7-1 Parallel Lines and Related Angles

What You'll Learn

- Identifying pairs of angles formed by two lines and a transversal
- Relating the measures of angles formed by parallel lines and a transversal

...And Why

To understand how parallel lines are used in building, city planning, and construction

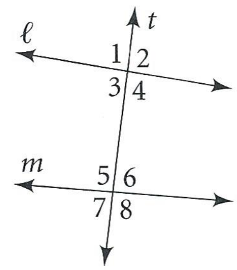
What You'll Need

- ruler
- protractor

THINK AND DISCUSS

Angles Formed by Intersecting Lines

A **transversal** is a line that intersects two coplanar lines at two distinct points. The diagram shows the eight angles formed by the transversal and the two lines.



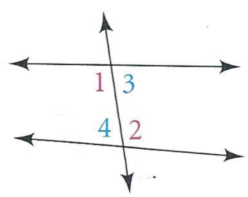
1. a. $\angle 3$ and $\angle 6$ are in the *interior* of ℓ and m. Name another pair of angles in the interior of ℓ and m.

b. Name a pair of angles in the *exterior* of ℓ and m.

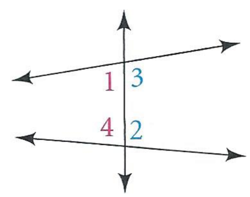
2. a. $\angle 1$ and $\angle 4$ are on *alternate sides* of the transversal t. Name another pair of angles on alternate sides of t.

b. Name a pair of angles on the *same side* of t.

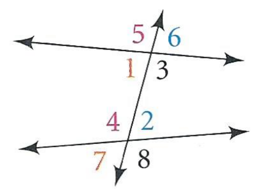
You can use the terms in Questions 1 and 2 to describe pairs of angles.



$\angle 1$ and $\angle 2$ are **alternate interior angles**, as are $\angle 3$ and $\angle 4$.

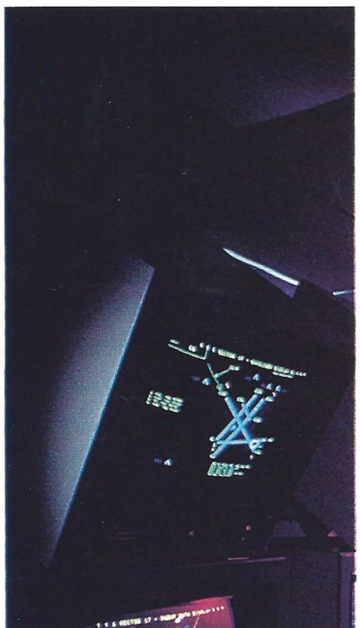


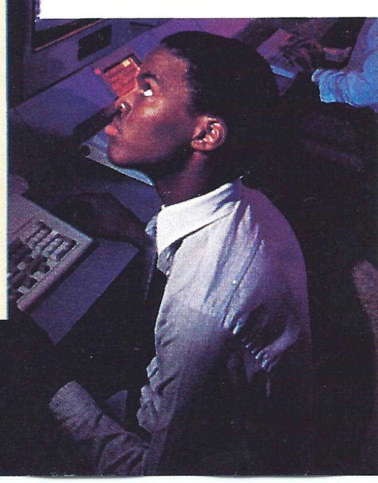
$\angle 1$ and $\angle 4$ are **same-side interior angles**, as are $\angle 2$ and $\angle 3$.

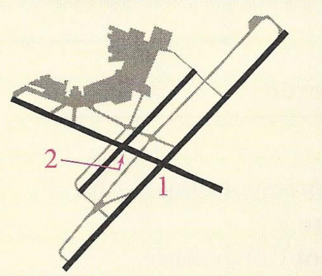


$\angle 5$ and $\angle 4$ are **corresponding angles**, as are $\angle 6$ and $\angle 2$, $\angle 1$ and $\angle 7$, and $\angle 3$ and $\angle 8$.

3. In the diagrams above, $\angle 1$ and $\angle 3$ are not alternate interior angles. What term decribes their positions in relation to each other?







Lafayette Regional Airport
Lafayette, Louisiana

Example 1 **Relating to the Real World**

Aviation In the diagram of Lafayette Regional Airport, the black segments are runways and the grey areas are taxiways and terminal buildings. Classify $\angle 1$ and $\angle 2$ as alternate interior angles, same-side interior angles, or corresponding angles.

$\angle 1$ and $\angle 2$ are corresponding angles.

WORK TOGETHER

Work with a partner.

- Draw two parallel lines using lined paper or the two edges of a ruler. Then draw a transversal that intersects the two parallel lines.
- Use a protractor to measure each of the eight angles formed. Record the measures on your drawing.

4. Make **conjectures** about the measures of corresponding angles, alternate interior angles, and same-side interior angles. Compare your results with those of your classmates.

TECHNOLOGY HINT

The Work Together could be done with geometry software.

THINK AND DISCUSS

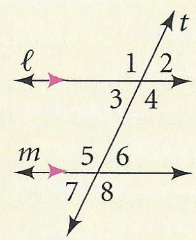
Angles Formed by Parallel Lines

The results of the Work Together lead to the following postulate and theorems.

Postulate 7-1
Corresponding Angles Postulate

If two parallel lines are cut by a transversal, then corresponding angles are congruent.

If $\ell \parallel m$, then $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, and $\angle 4 \cong \angle 8$.



Theorem 7-1
Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

If $\ell \parallel m$, then $\angle 3 \cong \angle 6$ and $\angle 4 \cong \angle 5$.

Theorem 7-2
Same-Side Interior Angles Theorem

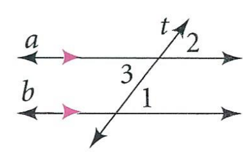
If two parallel lines are cut by a transversal, then the pairs of same-side interior angles are supplementary.

If $\ell \parallel m$, then $\angle 3$ and $\angle 5$ are supplementary, and so are $\angle 4$ and $\angle 6$.

Proof of Theorem 7-1

Given: $a \parallel b$

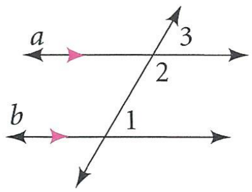
Prove: $\angle 1 \cong \angle 3$



Statements	Reasons
1. $a \parallel b$	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** If $\parallel$ lines, then corresponding $\angle$s are $\cong$.
3. $\angle 2 \cong \angle 3$	**3.** Vertical angles are $\cong$.
4. $\angle 1 \cong \angle 3$	**4.** Transitive Prop. of Congruence

When writing a proof, it is often helpful to start by writing a plan. The plan should describe how you can reason from the given information to what you want to prove.

Example 2

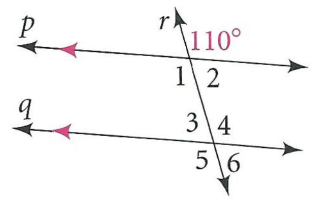


Write a plan for the proof of Theorem 7-2.

Given: $a \parallel b$

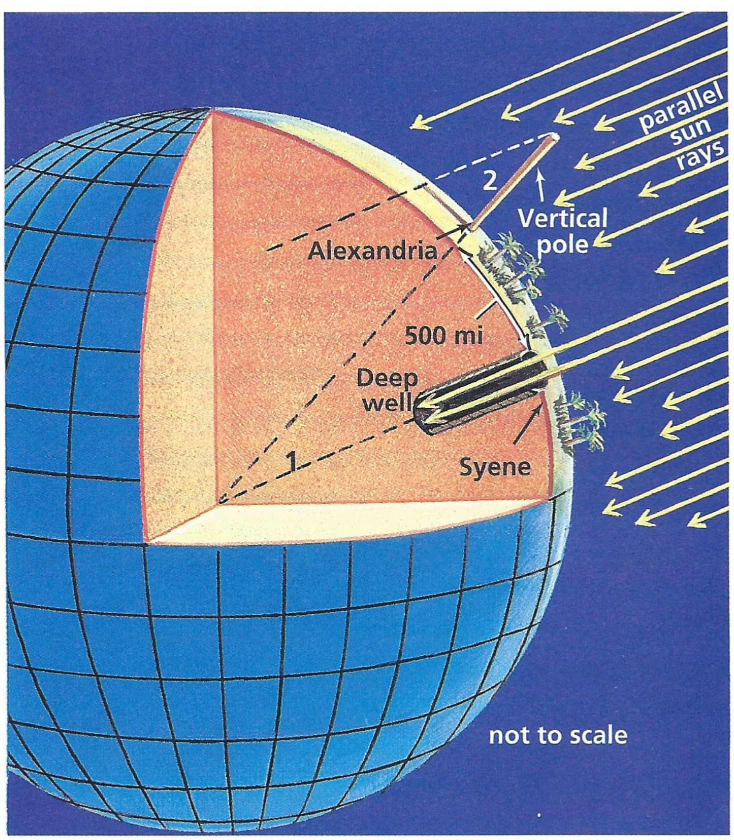
Prove: $\angle 1$ and $\angle 2$ are supplementary.

Plan for Proof To show that $m\angle 1 + m\angle 2 = 180$, show that $m\angle 3 + m\angle 2 = 180$ and that $m\angle 1 = m\angle 3$. Then substitute $m\angle 1$ for $m\angle 3$.



5. Use the Plan for Proof to write a two-column proof of Theorem 7-2.
6. **Try This** Find the measure of each angle.
   a. $\angle 1$ b. $\angle 2$ c. $\angle 3$
   d. $\angle 4$ e. $\angle 5$ f. $\angle 6$

Example 3 Relating to the Real World



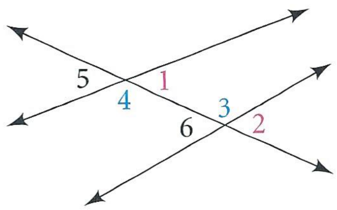
Measuring Earth About 220 B.C., Eratosthenes estimated the circumference of Earth. He achieved this remarkable feat by assuming that Earth is a sphere and that the sun's rays are parallel. He knew that the sun was directly over the town of Syene on the longest day of the year, because sunlight shone directly down a deep well. On that day, he measured the angle of the shadow of a vertical pole in Alexandria, which was 5000 stadia (about 500 miles) north of Syene. How did Eratosthenes know that $\angle 1 \cong \angle 2$? And how could he compute the circumference of Earth knowing $m\angle 1$?

Since $\angle 1$ and $\angle 2$ are alternate interior angles formed by the sun's parallel rays, the angles are congruent. The angle Eratosthenes measured was 7.2°. This is $\frac{1}{50}$ of 360°, so 500 mi is $\frac{1}{50}$ of the circumference of Earth. His estimate of 25,000 mi is very close to the actual value.

Exercises ON YOUR OWN

Classify each pair of angles labeled with the same color as alternate interior angles, same-side interior angles, or corresponding angles.

1.



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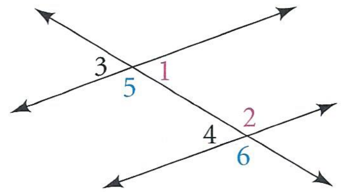

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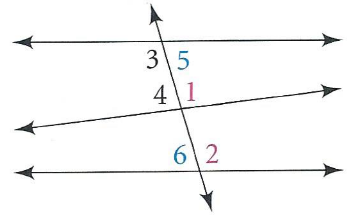
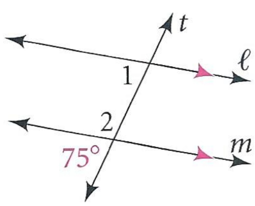


Find $m\angle 1$ and then $m\angle 2$. State the theorems or postulates that justify your answers.

4.



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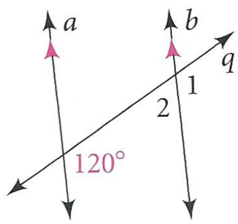

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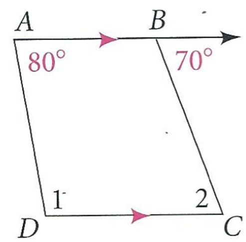

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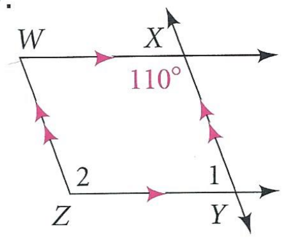
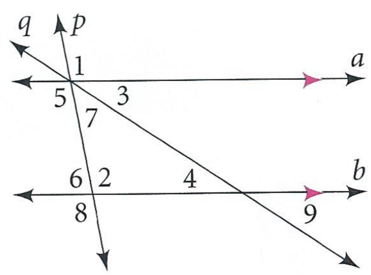


State the theorem or postulate that justifies each statement about the figure at the right.



8. $\angle 1 \cong \angle 2$

9. $\angle 3 \cong \angle 4$

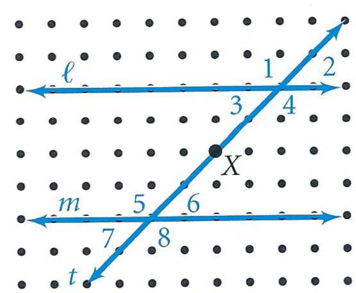
10. $m\angle 5 + m\angle 6 = 180$

11. $\angle 5 \cong \angle 2$

12. $\angle 5 \cong \angle 8$

13. $\angle 3 \cong \angle 9$

14. **Open-ended** The letter **Z** illustrates alternate interior angles. Find at least two other letters that illustrate the pairs of angles presented in this lesson. For each letter, show which types of angles are formed.



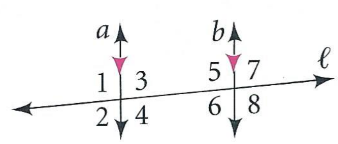
15. a. **Transformational Geometry** Lines ℓ and m are parallel, and line t is a transversal. Under the translation $\langle -4, -4 \rangle$, the image of each of the angles $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ is its __?__ angle.

b. Under a rotation of 180° in point X, the image of each of the angles $\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$ is its __?__ angle.

16. **Writing** Look up the meaning of the prefix *trans-*. Explain how the meaning of the prefix relates to the word *transversal*.

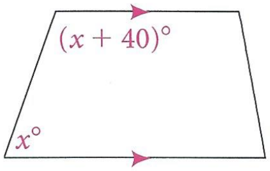
17. a. **Probability** Suppose that you pick one even-numbered angle and one odd-numbered angle from the diagram. Find the probability that the two angles are congruent.

b. **Open-ended** Write a probability problem of your own based on the diagram. Then solve it.



Algebra **Find the values of the variables.**

30.



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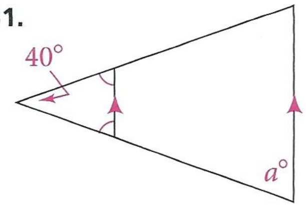

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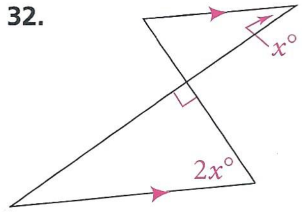

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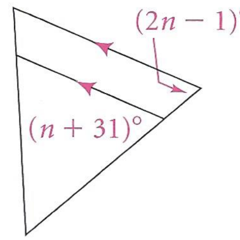

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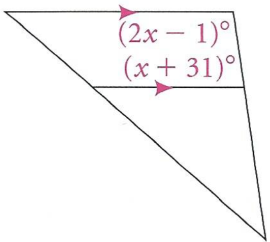

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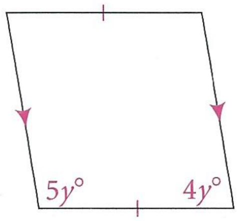

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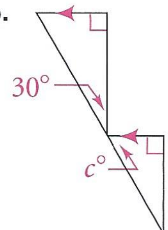

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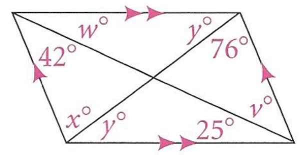
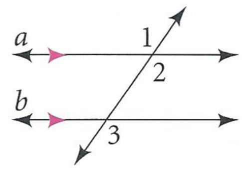


38. ∠1 and ∠3 are *alternate exterior angles.* Write a two-column proof of this statement: If two parallel lines are cut by a transversal, then alternate exterior angles are congruent.

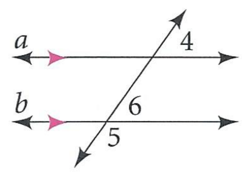


Given: $a \parallel b$

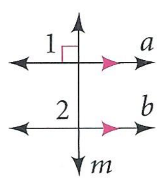
Prove: $\angle 1 \cong \angle 3$

Plan for Proof: Show that $\angle 1 \cong \angle 3$ by showing that both angles are congruent to ∠2.

39. **Critical Thinking** ∠4 and ∠5 are *same-side exterior angles.* Make a **conjecture** about the same-side exterior angles formed by two parallel lines and a transversal. Prove your conjecture.



40. Use the diagram to write a paragraph proof of this statement: If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line.



Given: $a \parallel b$, $m \perp a$

Prove: $m \perp b$

41. **Traffic Flow** You are designing the parking lot for a local shopping area, and are considering the two arrangements shown. Give the advantages and disadvantages of each.