What You'll Learn

- Calculating the surface areas and volumes of spheres

...And Why

To solve real-world problems such as finding the surface area of a soccer ball and the volume of a scoop of ice cream

What You'll Need

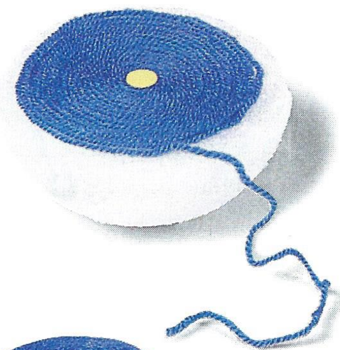
- ruler
- scissors
- tacks
- foam balls (cut in half)
- string
- calculator

6-6 Surface Areas and Volumes of Spheres

WORK TOGETHER

How do you compute the surface area of Earth or the amount of leather covering a baseball? Work in a group to explore the surface area of a sphere.

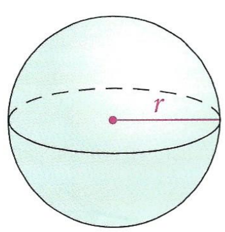
- The surface of half a foam ball consists of two parts—a plane circular region and a curved surface. Place a tack in the center of the circular region. Wind string around the tack covering the entire circular region.
- Once the circular region is covered, cut off any excess string. Measure the length x of string that covered the circular region.
- Place a tack in the center of the curved surface. Wind another string around the tack covering the entire curved surface.
- Once the curved surface is covered, cut off any excess string. Measure the length y of the string that covered the curved surface.



1. a. How do x and y compare?
   b. Express y as a multiple of x.
2. a. How much string would you need to cover the entire surface of an uncut foam ball? Express your answer as a multiple of y.
   b. Express your answer to part (a) as a multiple of x.
3. A string of length x covers an area of πr^2 where r is the radius of the foam ball. Substitute πr^2 for x in the expression you wrote for Question 2(b) to find a formula for the surface area of a sphere.

THINK AND DISCUSS

Finding the Surface Area of a Sphere



A **sphere** is the set of all points in space equidistant from a given point called the **center.**

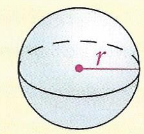
4. How would you define a *radius* of a sphere?
5. How would you define a *diameter* of a sphere?

Your discovery in the Work Together leads to the following theorem.

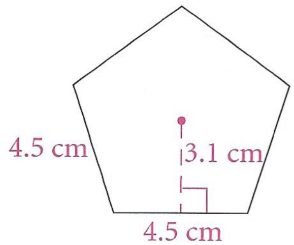
Theorem 6-10
Surface Area of a Sphere

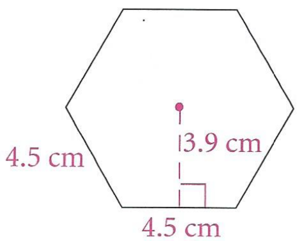
The surface area of a sphere is four times the product of π and the square of the radius of the sphere.

$$\text{S.A.} = 4\pi r^2$$



Example 1 *Relating to the Real World*





Manufacturing Manufacturers make soccer balls with a radius of 11 cm by sewing together 20 regular hexagons and 12 regular pentagons. Templates for guiding the stitching are shown at the left. Approximate the surface area of a soccer ball to the nearest square centimeter using the following two methods.

Method 1:
Find the sum of the areas of the pentagons and hexagons.

regular pentagon

$$\begin{aligned} A &= \tfrac{1}{2}ap \\ &= \tfrac{1}{2}(3.1)(5 \cdot 4.5) \\ &= 34.875 \end{aligned}$$

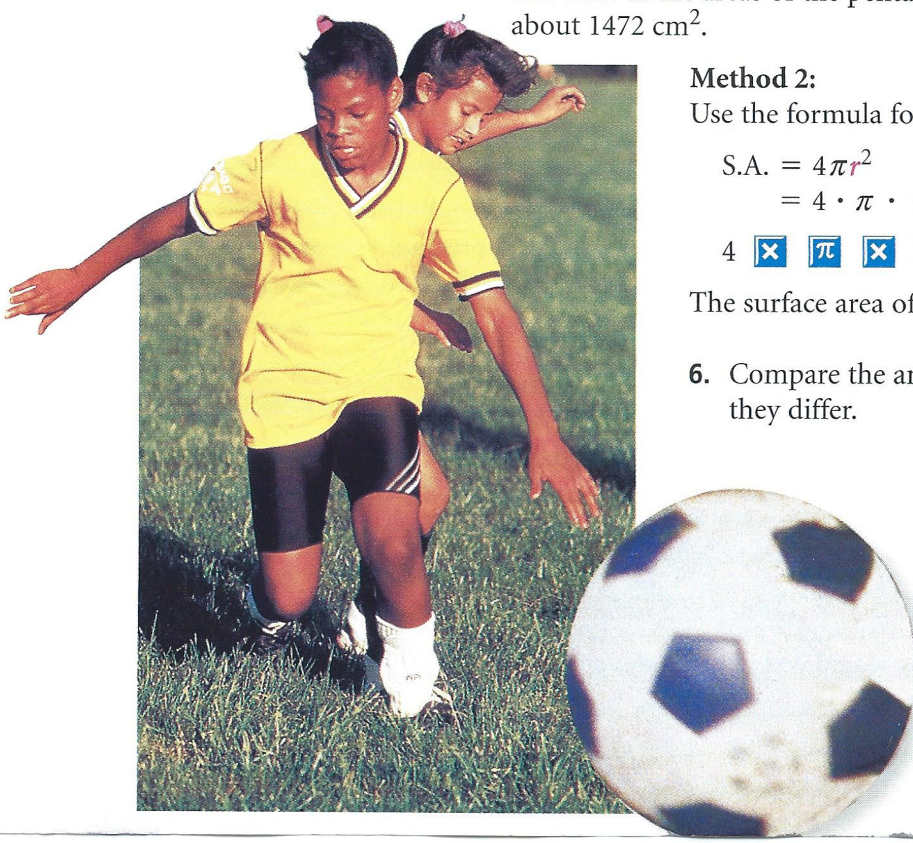
regular hexagon

$$\begin{aligned} A &= \tfrac{1}{2}ap \\ &= \tfrac{1}{2}(3.9)(6 \cdot 4.5) \\ &= 52.65 \end{aligned}$$

Area of the 12 regular pentagons $= (12)(34.875) = 418.5$

Area of the 20 regular hexagons $= (20)(52.65) = 1053$

The sum of the areas of the pentagons and the hexagons is about 1472 cm^2.



Method 2:
Use the formula for the surface area of a sphere.

$\text{S.A.} = 4\pi r^2$ **Use the formula for surface area.**

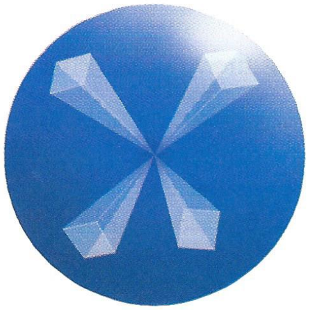
$= 4 \cdot \pi \cdot 11^2$ **Substitute.**

4 [×] [π] [×] 11 [x^2] [=] 1520.5308

The surface area of the soccer ball is about 1521 cm^2.

6. Compare the answers for the two methods. Explain why they differ.

Finding the Volume of a Sphere



You can fill a sphere with a large number n of small pyramids. The vertex of each pyramid is the center of the sphere. The height of each pyramid is approximately the radius r of the sphere. The sum of the areas of all the bases approximates the surface area of the sphere. You can use this model to derive a formula for the volume of a sphere.

Volume of each pyramid $= \frac{1}{3}Bh$

Sum of the volumes of n pyramids $= n \cdot \frac{1}{3}Br$ — **Substitute r for h.**

$= \frac{1}{3} \cdot (nB) \cdot r$

$= \frac{1}{3} \cdot (4\pi r^2) \cdot r$ — **Replace nB with the surface area of a sphere.**

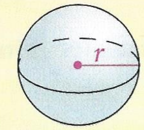
$= \frac{4}{3}\pi r^3$

The volume of a sphere is $\frac{4}{3}\pi r^3$.

Theorem 6-11
Volume of a Sphere

The volume of a sphere is four thirds the product of π and the cube of the radius of the sphere.

$$V = \frac{4}{3}\pi r^3$$



Example 2

The volume of a sphere is 4849.05 m^3. What is the surface area of the sphere? Round your answer to the nearest tenth.

- Find the radius r.

$V = \frac{4}{3}\pi r^3$ — **Use the formula for the volume of a sphere.**

$4849.05 = \frac{4}{3}\pi r^3$ — **Substitute.**

$4849.05\left(\frac{3}{4\pi}\right) = r^3$ — **Multiply both sides by $\frac{3}{4\pi}$.**

$\sqrt[3]{4849.05\left(\frac{3}{4\pi}\right)} = r$ — **Find the cube root of each side.**

4849.05 [×] 3 [÷] 4 [÷] [π] [=] [$\sqrt[x]{y}$] 3 [=] 10.500001

The radius of the sphere is about 10.5 m.

- Find the surface area of the sphere.

S.A. $= 4\pi r^2$ — **Use the formula for the surface area of a sphere.**

$= 4\pi(10.5)^2$ — **Substitute.**

4 [×] [π] [×] 10.5 [x^2] [=] 1385.4424

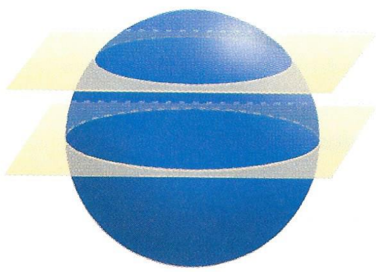
The surface area of the sphere is about 1385.4 m^2.

QUICK REVIEW

The *cube root* of x is the number that when *cubed* is x.

7. **Try This** The volume of a sphere is 20,579 in.^{3}. What is the radius of the sphere to the nearest whole number?

8. **Try This** The radius of a sphere is 15 m. What is the volume to the nearest hundred?



When a plane and a sphere intersect in more than one point, the intersection is a circle. If the center of the circle is also the center of the sphere, the circle is called a **great circle** of the sphere. The circumference of a great circle is the **circumference of the sphere.** A great circle divides a sphere into two **hemispheres.**

9. **Geography** What is the name of the best-known great circle on Earth?

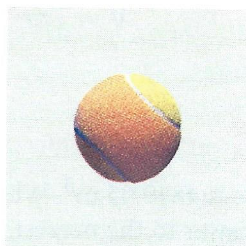
10. **Geography** Describe the Northern Hemisphere of Earth.

Exercises ON YOUR OWN

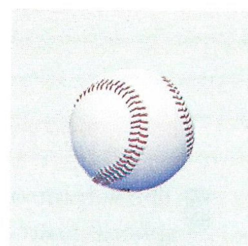
Calculator **Find the surface area of each ball to the nearest tenth.**

1. 

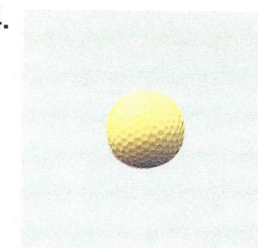
$d = 23.9$ cm

2. 

$d = 68$ mm

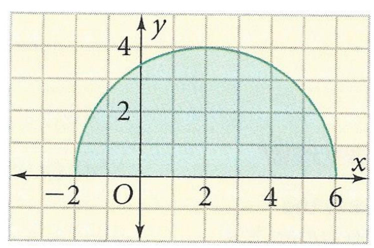
3. 

$d = 2\frac{3}{4}$ in.

4. 

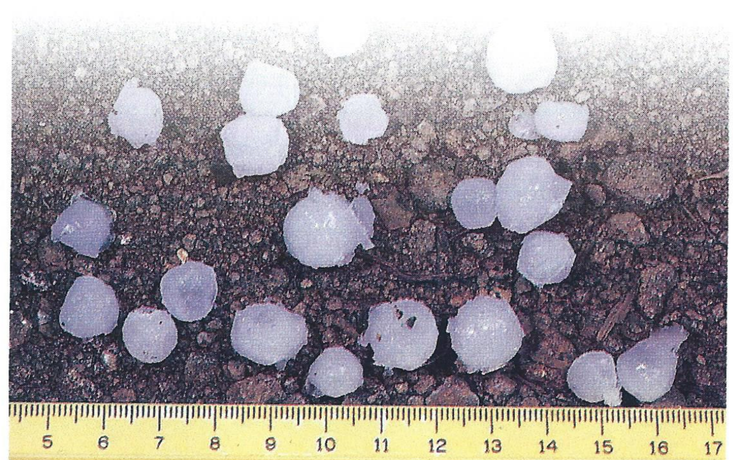
$d = 1.68$ in.

5. **Coordinate Geometry** Find the surface area and volume of the sphere formed by rotating the semicircle at the right 360° about the x-axis. Leave your answers in terms of π.

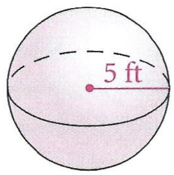


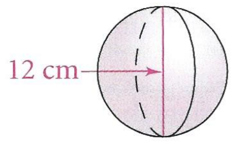
6. A balloon has a 14-in. diameter when it is fully inflated. Half the air is let out of the balloon. Assuming the balloon is a sphere, what is the new diameter? Round your answer to the nearest inch.

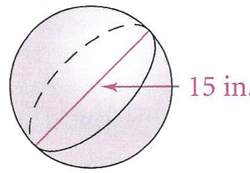
7. **Meteorology** On June 16, 1882, a massive thunderstorm over Dubuque, Iowa, produced huge hailstones. The circumference of some of the hailstones was 17 in. Ice weighs about 0.033 lb/in.^{3}. What was the approximate weight of these hailstones to the nearest tenth of a pound?

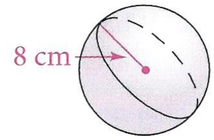


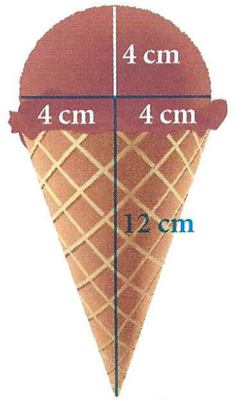
Find the volume of each sphere. Leave your answers in terms of π.

8. 

9. 

10. 

11. 



12. **Sports** The circumference of a bowling ball is about 27 in. Find its volume to the nearest tenth.

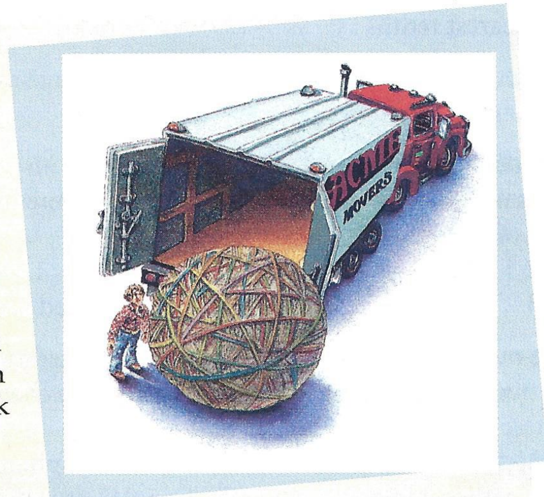
13. If the sphere of ice cream shown melts, is the cone large enough to hold the melted ice cream? Explain.

14. **Geometry in 3 Dimensions** The center of a sphere has coordinates (0, 0, 0). The radius of the sphere is 5.
    a. Name the coordinates of six points on the sphere.
    b. Tell whether each of the following points is inside, outside, or on the sphere. (*Hint:* Use the formula from page 262.)
    $A(0, -3, 4)$, $B(1, -1, -1)$, $C(4, -6, -10)$

Believe It Or Not

J.C. Payne, a Texas farmer, is the world champion string collector. The ball of string he wound over a three-year period has a circumference of 41.5 ft. It weighed 13,000 lb.

Listed in the Guinness Book of Records, the ball of string is now on display in a museum devoted to oddities. It took almost a dozen men with forklift trucks to load the ball onto a truck to move it to the museum.



15. a. What is the volume of the ball of string to the nearest cubic foot?
    b. What is the weight of the string per cubic foot?
    c. If the diameter of the string is 0.1 in., what is the approximate length of the string to the nearest mile? (*Hint:* Think of the unwound string as a long cylinder.)

CALCULATOR HINT

When answers will be used in later calculations, keep or store them in unrounded form so that rounding errors will not be introduced into the final answer.

16. The radius of Earth is approximately 3960 mi. The area of Australia is about 2,940,000 mi^2.
    a. Find the surface area of the Southern Hemisphere.
    b. **Probability** If a meteorite falls randomly in the Southern Hemisphere, what is the probability that it will fall in Australia?