6-7 Composite Space Figures

What You'll Learn

- Recognizing composite space figures, which combine two or more simple figures

...And Why

To find the volumes and surface areas of composite space figures such as silos and backpacks

What You'll Need

- unit cubes

WORK TOGETHER

1. Work in a group to build the prisms shown in Figures 1 and 2 using unit cubes. Find the surface area and volume of each prism.
2. What is the sum of the surface areas of the two prisms? What is the sum of their volumes?
3. Place one prism on top of the other as in Figure 3. Find the surface area and volume of the resulting space figure.
4. Compare your answers to Questions 2 and 3. How does the relationship of the volumes differ from that of the surface areas?

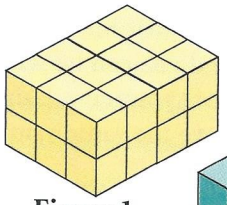


Figure 1

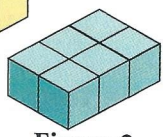


Figure 2

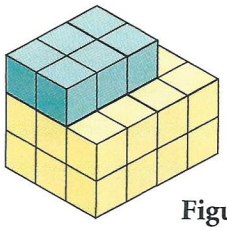
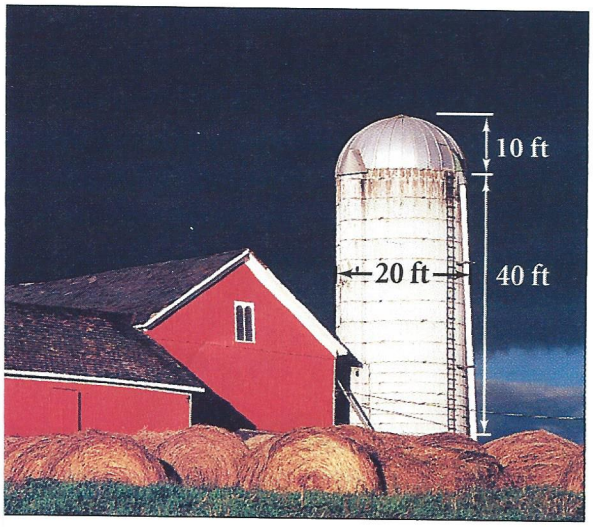


Figure 3

THINK AND DISCUSS

You can use what you know about the volumes of three-dimensional figures such as prisms, pyramids, cones, cylinders, and spheres to find the volume of a composite space figure. A **composite space figure** combines two or more of these figures. The volume of a composite space figure is the sum of the volumes of the figures that are combined.



Example 1 Relating to the Real World

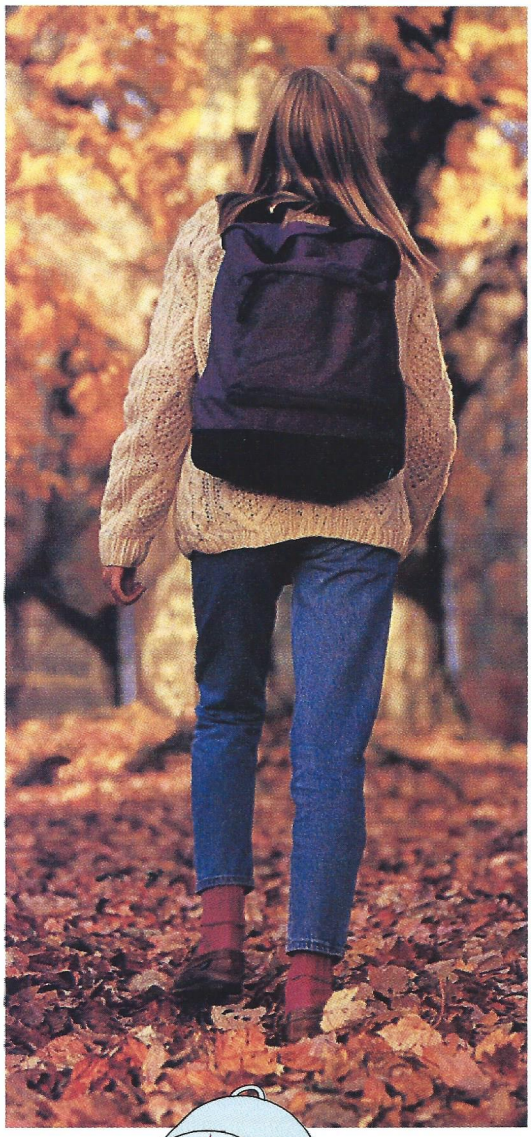
Agriculture Find the volume of the grain silo at the left.

The silo combines a cylinder and a hemisphere.

- Volume of the cylinder $= \pi r^2 h = \pi (10)^2 (40) = 4000\pi$
- Volume of the hemisphere $= \frac{1}{2}(\frac{4}{3}\pi r^3) = \frac{2}{3}\pi (10)^3 = \frac{2000\pi}{3}$
- Volume of the composite figure $= 4000\pi + \frac{2000\pi}{3}$

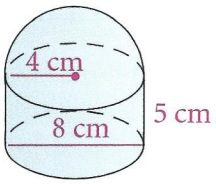
4000 × π + 2000 × π ÷ 3 = 14660.766

The volume of the silo is about 14,700 ft^3.



5. Try This Find the volume of each composite space figure to the nearest whole number.

a.



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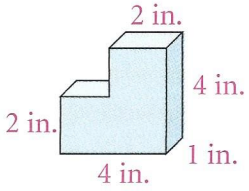
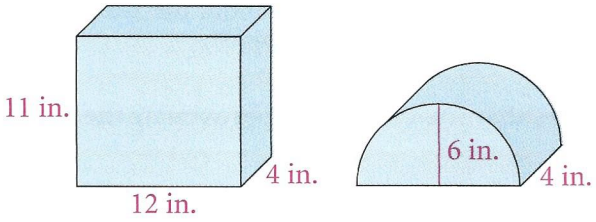


You can use geometric figures to approximate the shape of a real-world object. Then you can estimate the volume and surface area of the object.

Example 2 Relating to the Real World

Estimation What space figures can you use to approximate the shape of the backpack? Use these space figures to estimate the volume of the backpack.

- You can use a prism and half of a cylinder to approximate the shape of the backpack.



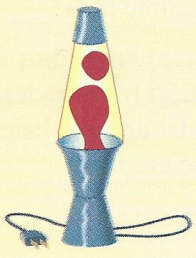
- Volume of the prism $= Bh = (12 \cdot 4)11 = 528$
- Volume of the half cylinder $= \frac{1}{2}(\pi r^2 h) = \frac{1}{2}\pi(6)^2(4)$
  $= \frac{1}{2}\pi(36)(4) \approx 226$
- Sum of the two volumes $= 528 + 226 = 754$

The approximate volume of the backpack is 754 in.^{3}.

6. What is the approximate surface area of this backpack?

7. Try This Describe the space figures that you can use to approximate the shape of each object.

a.



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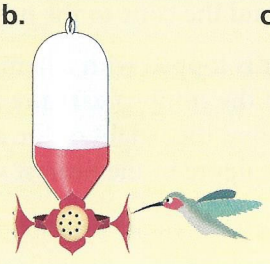
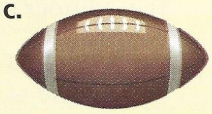
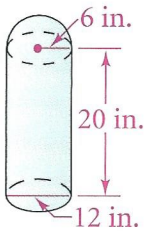


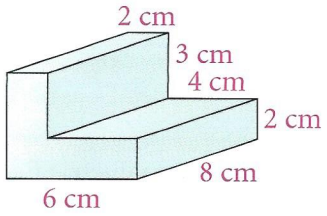
c.

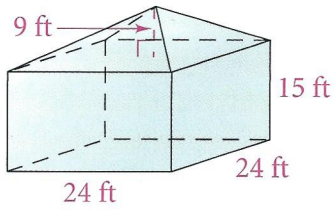


Exercises ON YOUR OWN

Find the volume of each composite space figure. You may leave answers in terms of π.

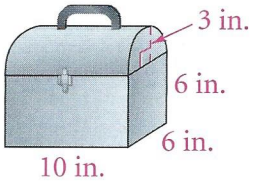
1. 

2. 

3. 

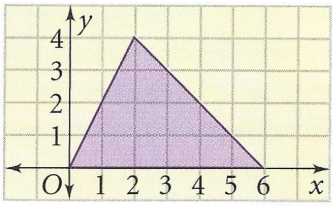
4. **Open-ended** Draw a composite three-dimensional figure, label its dimensions, and find either its surface area or its volume.

5. **Manufacturing** Find the volume of the lunch box shown at the right to the nearest cubic inch.

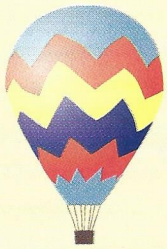


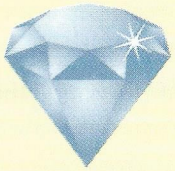
6. **Writing** Describe your home, school, or some other building that is a composite space figure. Explain why it is a composite space figure.

7. **a. Coordinate Geometry** Draw a sketch of the composite space figure formed by rotating this triangle 360° about the x-axis.
   b. Find the volume of the figure in terms of π.



Describe space figures that you can use to approximate the shape of each object.

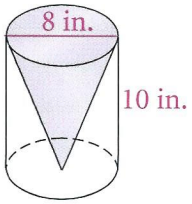
8. 

9. 

10. 

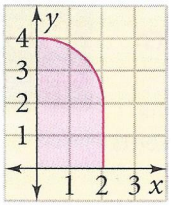
11. **Writing** Describe how you would find the volume of an octahedron if you know the length of an edge. (See the picture on page 305.)

12. In the diagram at the right, what is the volume of the space between the cylinder and the cone to the nearest cubic inch?

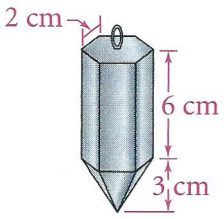


13. **a.** A cylinder is topped with a hemisphere with radius 15 ft. The total volume of the composite figure is 6525π ft^3. Sketch the figure.
   b. Algebra Find the height of the cylinder and the height of the composite figure to the nearest whole number.

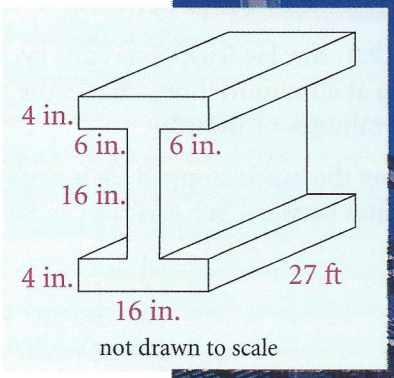
14. Coordinate Geometry Find the surface area of the figure formed by rotating the figure at the right 360° about the y-axis. Leave your answer in terms of π.



15. Carpentry Builders use a plumb bob to establish a vertical line. To the nearest cubic centimeter, find the volume of the plumb bob shown at the right. It combines a hexagonal prism with a pyramid.



Engineering **Steelworkers use I-beams like the one shown to build bridges and overpasses.**



16. To the nearest cubic foot, find the volume of steel needed to make the beam.

17. a. To the nearest square foot, what is the surface area of the beam?

b. One gallon of paint covers about 450 square feet. How many 1-gallon cans of paint do you need to paint the beam?

Exercises MIXED REVIEW

Write an indirect proof for each statement.

18. If a triangle is isosceles, then a base angle is not a right angle.

19. In $\triangle MNP$, if $MP < MN$, then $\angle P \ncong \angle N$.

20. Coordinate Geometry Find the circumference of a circle if the endpoints of a diameter are (3, 7) and (3, −1). Leave your answer in terms of π.

FOR YOUR JOURNAL

Describe an object in your classroom that is a composite space figure. Explain how you can approximate its volume.

Getting Ready for Lesson 6-8

You roll a number cube. Find each probability.

21. $P(4)$

22. P(odd number)

23. P(prime number)

24. P(2 or 5)