6-5   Volumes of Pyramids and Cones

What You'll Learn

- Finding volumes of pyramids and cones

...And Why

To solve problems concerning the amount of space in things such as a convention center or a popcorn box
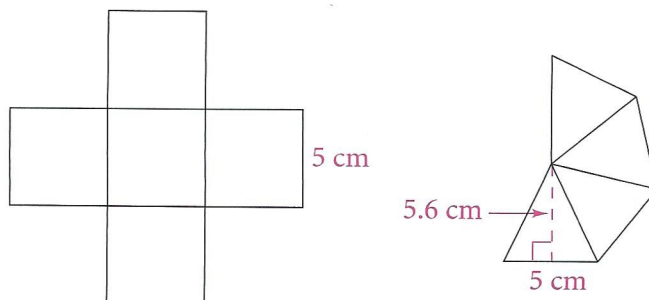
What You'll Need

- cardboard
- ruler
- scissors
- tape
- rice
- calculator

WORK TOGETHER

You know how to find the volume of a prism. Work in a group to explore the volume of a pyramid.

- Draw the nets shown below on cardboard.

- Cut out the nets and tape them together to make a cube and a regular square pyramid. Each model will have one open face.

5 cm

5.6 cm

5 cm

1. Compare the areas of the bases of the cube and the pyramid.

2. Compare the heights of the cube and the pyramid.

3. Fill the pyramid with rice. Then pour the rice from the pyramid into the cube. How many pyramids full of rice does the cube hold?

4. The volume of the pyramid is what fractional part of the volume of the cube?
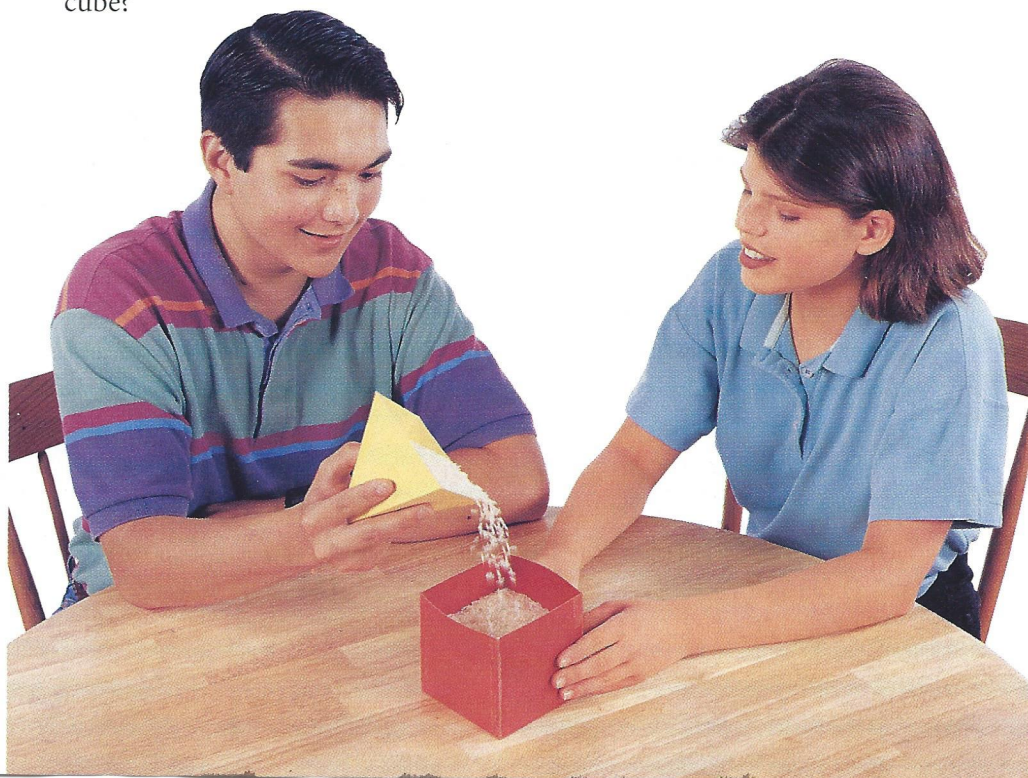
Volumes of Pyramids
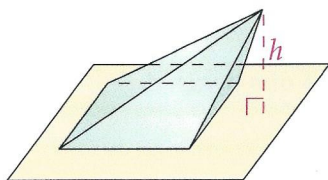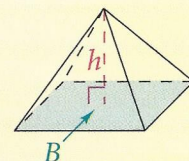
The Work Together demonstrates the following theorem.

| **Theorem 6-8**
Volume of a Pyramid | The volume of a pyramid is one third the product of the area of the base and the height of the pyramid.

$$V = \frac{1}{3}Bh$$ |

Oblique Pyramid

Because of Cavalieri's Principle, the volume formula is true for all pyramids, including *oblique* pyramids. The **height** h of an oblique pyramid is the length of the perpendicular segment from the vertex to the plane of the base.

Example 1   Relating to the Real World 🌐

Architecture   The Pyramid is an arena in Memphis, Tennessee. The area of the base of the Pyramid is about 300,000 ft². Its height is 321 ft. What is the volume of the Pyramid?

$V = \frac{1}{3}Bh$ **Use the formula for the volume of a pyramid.**
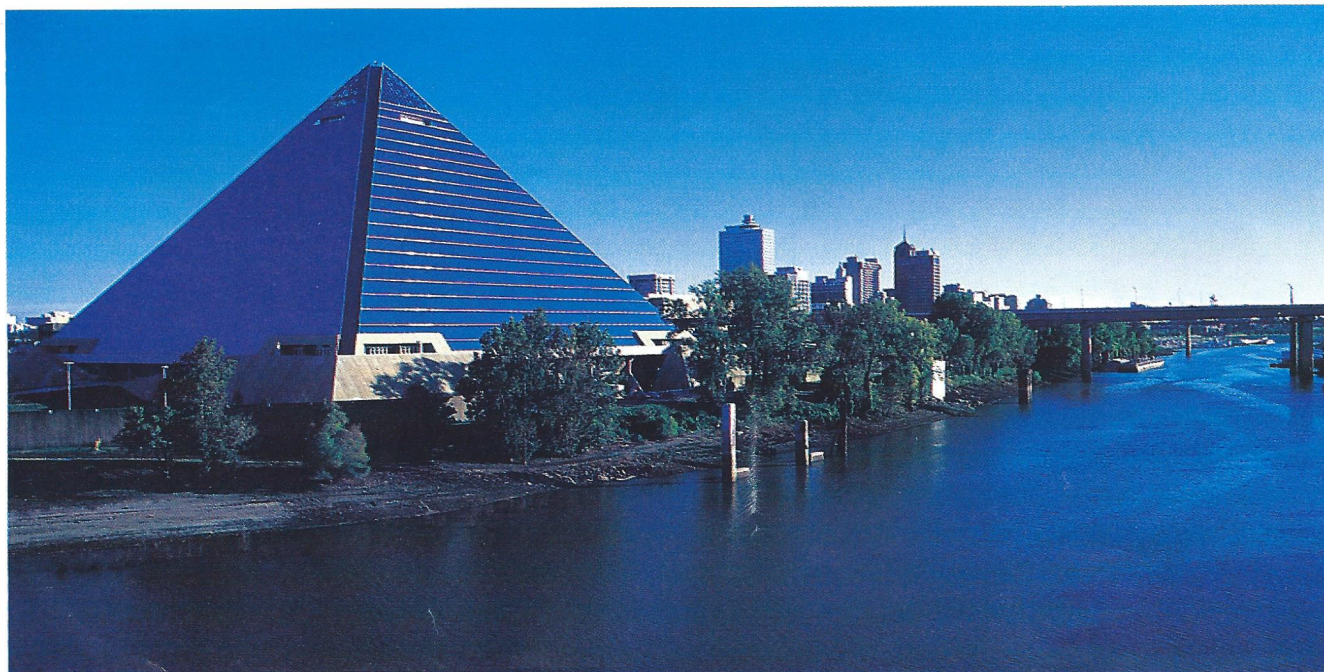
$ = \frac{1}{3}(300,000)(321)$ **Substitute.**

$ = 32,100,000$ **Simplify.**
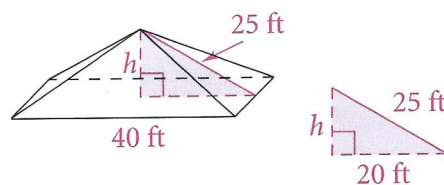
The volume is about 32,100,000 ft³.

5. **Try This** Find the volume of a regular square pyramid with base edges 12 in. long and height 8 in.

To find the volume of a pyramid you need to know its height.

Example 2

Find the volume of a regular square pyramid with base edges 40 ft long and slant height 25 ft.

25 ft

h

40 ft

25 ft

h

20 ft

- Find the height of the pyramid.

$$25^2 = h^2 + 20^2$$ **Use the Pythagorean Theorem.**
$$625 = h^2 + 400$$ **Simplify.**
$$h^2 = 225$$ **Subtract 400 from each side.**
$$h = 15$$ **Find the square root of each side.**

- Find the volume of the pyramid.

$$V = \tfrac{1}{3}Bh$$ **Use the formula for volume of a pyramid.**
$$= \tfrac{1}{3}(40 \cdot 40)15$$ **Substitute.**
$$= 8000$$ **Simplify.**

The volume of the pyramid is 8000 ft^3.

6. **Try This** Find the volume of a regular square pyramid with base edges 24 m long and slant height 13 m.

Volumes of Cones

In the Work Together, you discovered that the volume of a pyramid is one third the volume of a prism with the same base and height. You can also find that the volume of a cone is one third the volume of a cylinder with the same base and height.

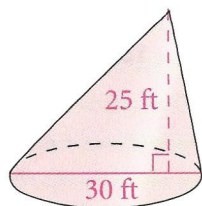| **Theorem 6-9** **Volume of a Cone**	The volume of a cone is one third the product of the area of the base and the height. $$V = \tfrac{1}{3}Bh, \text{ or } V = \tfrac{1}{3}\pi r^2 h$$	h B
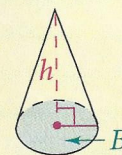
This volume formula applies to all cones, including *oblique* cones.

Example 3

Find the volume of the oblique cone with diameter 30 ft and height 25 ft. Round to the nearest whole number.

$r = \frac{30}{2} = 15$ **The radius is half the diameter.**

$V = \frac{1}{3}\pi r^2 h$ **Use the formula for the volume of a cone.**
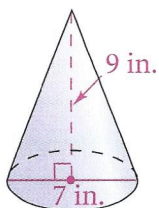
$= \frac{1}{3}\pi(15)^2 25$ **Substitute.**

1 ÷ 3 × π × 15 x² × 25 = 5890.4862

The volume of the cone is about 5890 ft³.

7. **Try This** Find the volume of a cone with radius 3 in. and height 8 in. Round to the nearest tenth.

Exercises   O N   Y O U R   O W N

Calculator Find the volume of each figure. When an answer is not a whole number, round to the nearest tenth.
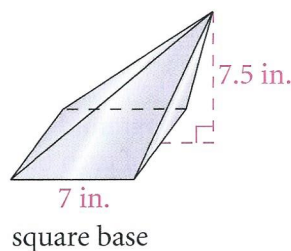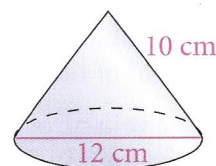
1.

9 in.
7 in.

2.

$5\frac{1}{2}$ in.
4 in.

3.

7.5 in.
7 in.
square base

4.

10 cm
12 cm
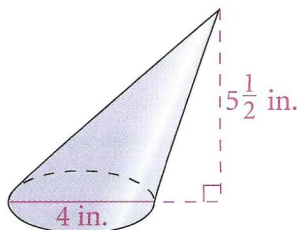
5. a. **Mental Math** A cone with radius 3 ft and height 10 ft has a volume of 30π ft³. What is its volume when the radius is doubled?
   b. What is the volume when the height of the original cone is doubled?
   c. What is the volume when both the radius and the height of the original cone are doubled?

6. a. The largest tepee in the United States belongs to a member of the Crow (Native Americans of the Great Plains). It is 43 ft high and 42 ft in diameter. Find its volume to the nearest cubic foot.
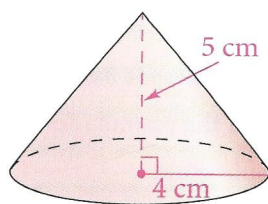   b. How does this compare with the volume of your classroom?
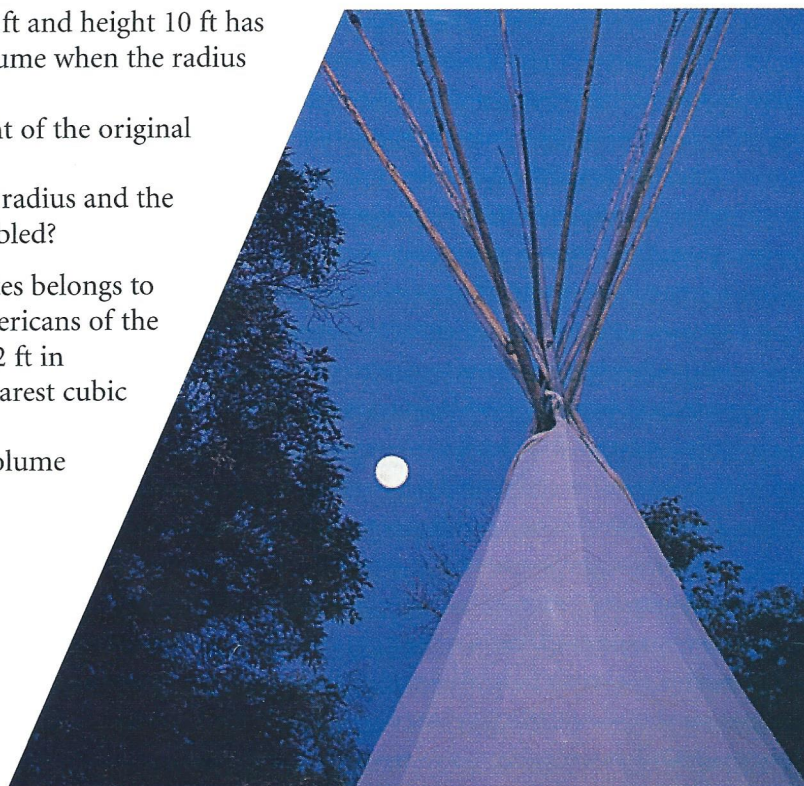
Find the volume of each figure. You may leave answers in terms of π or in simplest radical form.
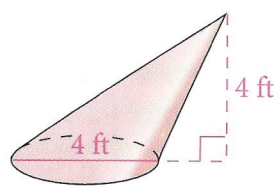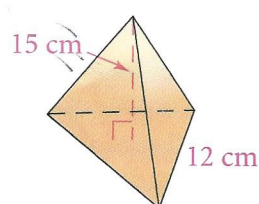
7.

9 in.

10 in.

8.

5 cm

4 cm

9.

4 ft

4 ft

10.

15 cm

12 cm
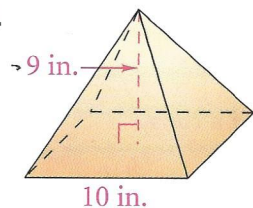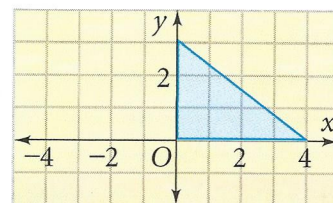
11. **Open-ended** A cone has a volume of 600π in.^{3}. Find two possible sets of dimensions for the height and radius of the cone.

12. a. **Coordinate Geometry** Suppose you rotate the right triangle shown 360° about the x-axis. What is the volume of the resulting cone in terms of π?

   b. Suppose you rotate the triangle 360° about the y-axis. What is the volume in terms of π?

13. The two cylinders pictured at the right are congruent. How does the volume of the larger cone compare to the total volume of the two smaller cones? Explain.
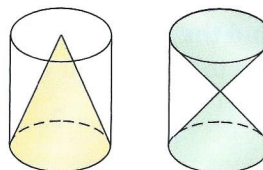
14. The volume of a cone is 36π cm^3. If the radius is 6 cm, what is the height of the cone?

15. The volume of a regular square pyramid is 600 in.^{3}. The height is 8 in. What is the length of each base edge?

16. To the nearest tenth, find the volume of a regular hexagonal pyramid with base edges 12 cm long and height 15 cm.

17. a. **Architecture** The Transamerica Building in San Francisco is a pyramid 800 ft tall with a square base 149 ft on each side. What is its volume to the nearest thousand cubic feet?

   b. If the Transamerica Building had been built as a *prism* with the same square base, how tall to the nearest foot would it have to be to have the same volume as the existing building?
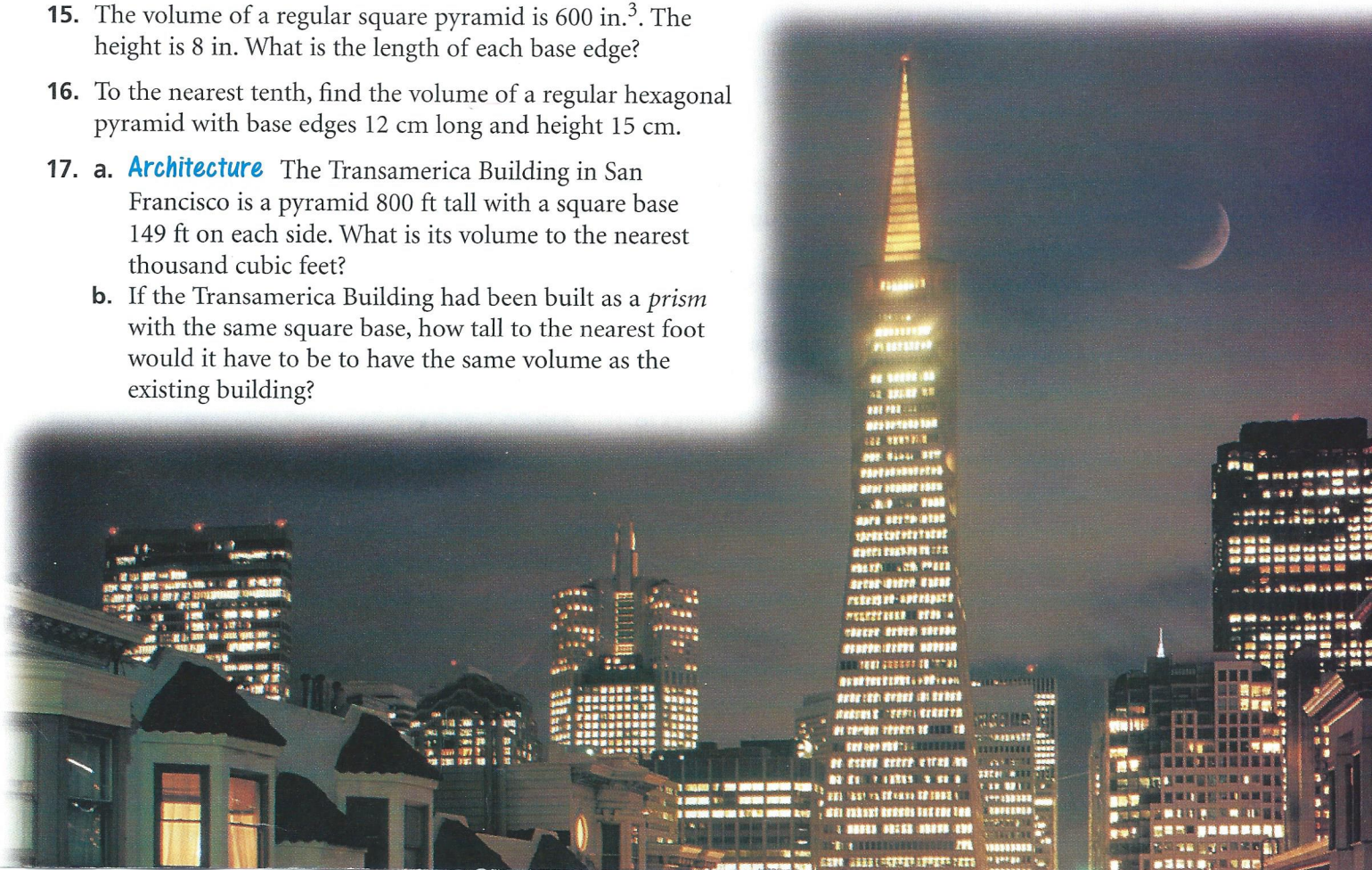
18. **Critical Thinking** A movie theater sells popcorn in cylindrical containers that are 4 in. in diameter and 10 in. high. As a special promotion, the theater plans to sell popcorn in a cone-shaped container for the same price. The diameter of the container will remain the same. The promotional cone will use the same amount of cardboard as the cylindrical container. Do you think this promotional cone is a good value? Why or why not?
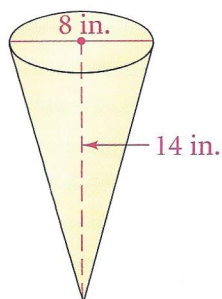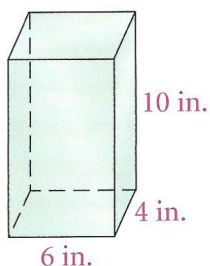
19. Which container has the greatest volume?
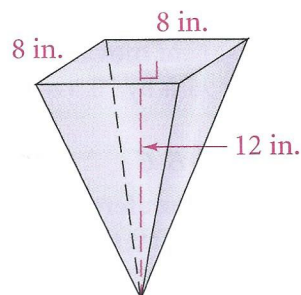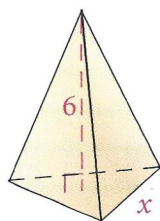

Figure 1   Figure 2   Figure 3

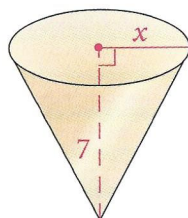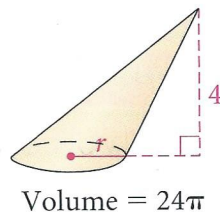**Algebra** Find the value of the variable in each figure. You may leave answers in simplest radical form.
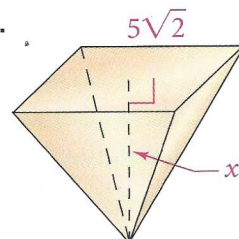
20.

Volume = $18\sqrt{3}$

21.

Volume = 21π

22.

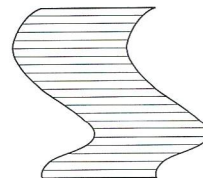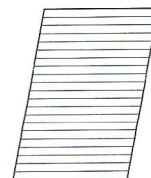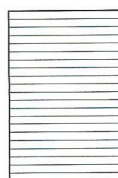Volume = 24π

23.

Volume = 150

24. **Writing** The figures at the right can be covered by an equal number of straws that are the same length. Describe how Cavalieri's Principle could be adapted to compare the areas of these figures.

Chapter Project ..... **Find Out by Investigating**

Do some container shapes seem to give more product for the same money? Go to a supermarket and identify a variety of container shapes. Do some shapes make you think there's more in them than there actually is? What factors do you think a manufacturer considers in deciding the shape of a container? Write a report about your findings.

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