6-4 Volumes of Prisms and Cylinders

What You'll Learn
- Finding the volumes of prisms and cylinders

...And Why
To find the space needed to store packages and the amount of water needed to fill an aquarium

What You'll Need
- unit cubes
- calculator

WORK TOGETHER

Work with a group. Explore the volume of a prism with unit cubes.

1. Make a single-layer rectangular prism that is 4 cubes long and 2 cubes wide. The prism will be 4 units by 2 units by 1 unit. How many cubes are in the prism?

2. Add a second layer to your prism to make a prism 4 units by 2 units by 2 units. How many cubes are in this prism?

3. Add a third layer to your prism to make a prism 4 units by 2 units by 3 units. How many cubes are in this prism?

4. How many cubes would be in the prism if you added two additional layers of cubes for a total of 5 layers?

5. How many cubes would be in the prism if there were 10 layers?

6. Compare a prism 2 cubes long by 3 cubes wide and 4 layers high with the prism in Question 3. What do you notice?

7. Compare a prism 3 cubes long by 4 cubes wide and 2 layers high with the prisms in Questions 3 and 6. What do you notice?
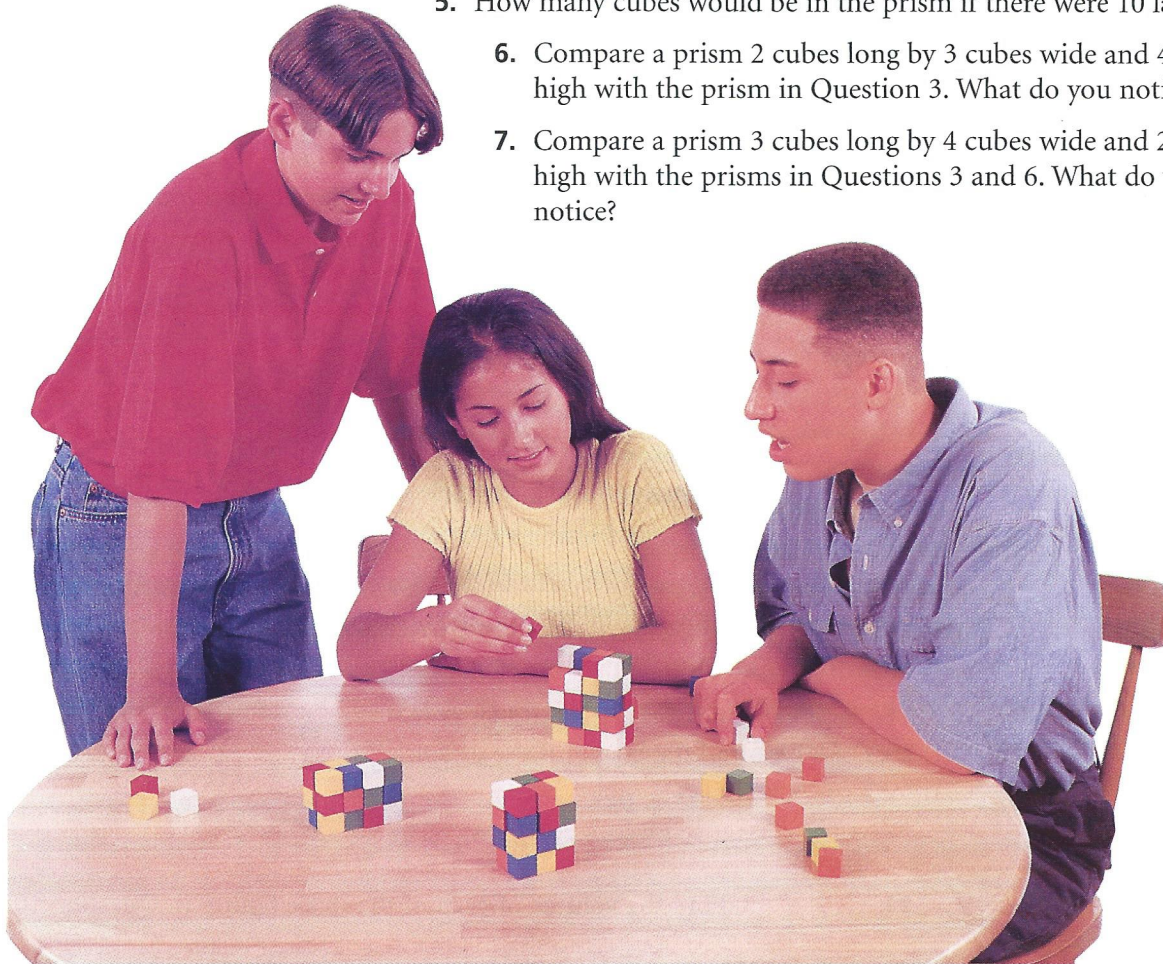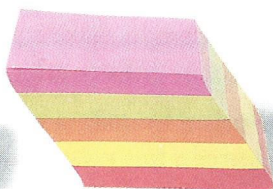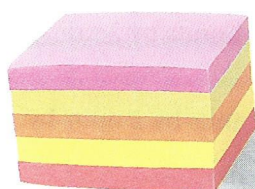
Volumes of Prisms
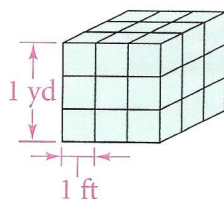
Volume is the space that a figure occupies. It is measured in cubic units such as cubic inches (in.^{3}), cubic feet (ft^3), or cubic centimeters (cm^3). The volume of a cube is the cube of the length of its edge, or $V = e^3$.

8. How many cubic feet are in a cubic yard?

**9.** How many cubic inches are in a cubic foot? in a cubic yard?

Each of the two stacks of paper at the left contains the same number of sheets. The first stack forms a right prism; the second forms an oblique prism. The stacks have the same height. The area of every cross section parallel to a base is the area of one sheet of paper. The stacks have the same volume. These stacks illustrate the following principle.

Theorem 6-5
Cavalieri's Principle

If two space figures have the same height and the same cross-sectional area at every level, then they have the same volume.

10. a. Try This What are the areas of the shaded cross sections of the space figures below?

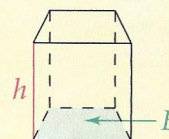**b.** Each of these figures has a height of 7 cm. What do you know about the volumes of these figures?
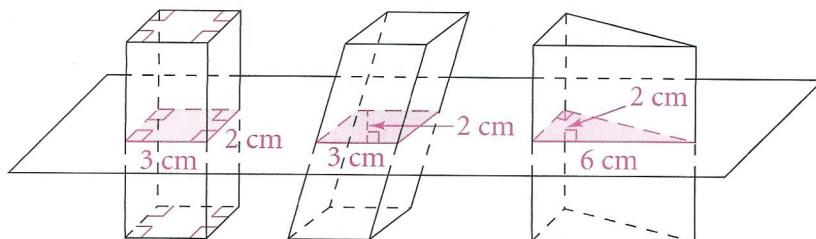
You can use what you discovered in the Work Together, along with Cavalieri's Principle, to find the volume of any prism.

Theorem 6-6
Volume of a Prism

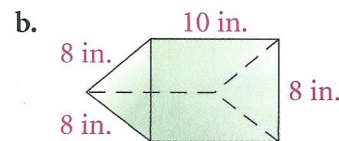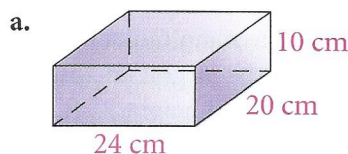The volume of a prism is the product of the area of a base and the height of the prism.

$$V = Bh$$

11. Try This Find the volume of each of the space figures in Question 10.
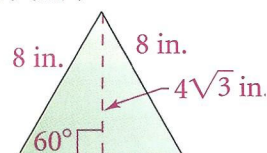
Example 1

Find the volume of each prism.

a.

10 cm
20 cm
24 cm

b.

10 in.
8 in.
8 in.
8 in.

a. $V = Bh$ **Use the formula for volume.**

$\quad = 480 \cdot 10$ $B = 24 \cdot 20 = 480 \text{ cm}^2$

$\quad = 4800$ **Simplify.**

The volume of the rectangular prism is 4800 cm^3.

b. The base of the triangular prism is an equilateral triangle. An altitude of the triangle divides it into two 30°-60°-90° triangles.

8 in.
8 in.
$4\sqrt{3}$ in.
60°

$\quad V = Bh$ **Use the formula for volume.**

$\quad\quad = 16\sqrt{3} \cdot 10$ $B = \frac{1}{2} \cdot 8 \cdot 4\sqrt{3} = 16\sqrt{3} \text{ in.}^2$

$\quad\quad = 160\sqrt{3}$ **Simplify.**

The volume of the triangular prism is $160\sqrt{3}$ in.^{3}.

12. **Try This** The volume of a triangular prism is 1860 cm^3. Its base is a right triangle with legs 24 cm and 10 cm long.
   a. Draw and label a diagram.
   b. Find the area of the base of the prism.
   c. Find the height of the prism.

13. **Try This** Find the volume of this oblique prism with rectangular bases.
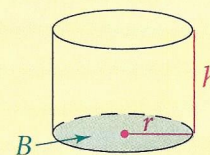

12 in.
11 in.
5 in.
10 in.
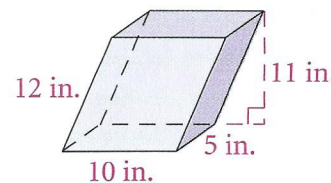
Volumes of Cylinders

The formula for the volume of a cylinder is similar to the formula for the volume of a prism.
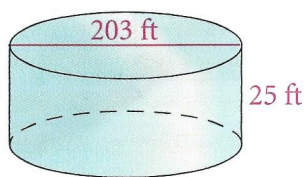
Theorem 6-7 **Volume of a Cylinder**	The volume of a cylinder is the product of the area of a base and the height of the cylinder. $$V = Bh, \text{ or } V = \pi r^2 h$$	 h B r

Sometimes you need to convert units of measure to solve a problem.

Example 2 **Relating to the Real World**

Aquarium Tanks The main tank at the Living Seas Aquarium at EPCOT Center in Florida is the largest enclosed tank in the world. It is a cylinder with diameter 203 ft and height 25 ft. About how many million gallons of water does this tank hold? (1 gal $\approx$ 231 in.^{3})

- Find the volume of the tank in cubic feet.

$r = \frac{203}{2} = 101.5$ **The radius equals half the diameter.**

$V = \pi r^2 h$ **Use the formula for the volume of a cylinder.**

$V = \pi (101.5)^2 (25)$ **Substitute.**

$\boxed{\pi}$ $\boxed{\times}$ 101.5 $\boxed{x^2}$ $\boxed{\times}$ 25 $\boxed{=}$ *809136.8229*

- To convert cubic feet to cubic inches, multiply by 12^3.

809,136.8229 $\boxed{\times}$ 12 $\boxed{y^x}$ 3 $\boxed{=}$ *1398188430*

- To find the number of gallons the tank can hold, divide by 231.

1,398,188,430 $\boxed{\div}$ 231 $\boxed{=}$ *6052763.766*

The main tank can hold about 6 million gallons of water.

14. The main tank at the New England Aquarium in Boston, Massachusetts, has a diameter of 40 ft and is 23 ft deep.

  a. What is the capacity of this tank? Round your answer to the nearest thousand gallons.

  b. How does the size of this tank compare with the size of the one at the Living Seas Aquarium?
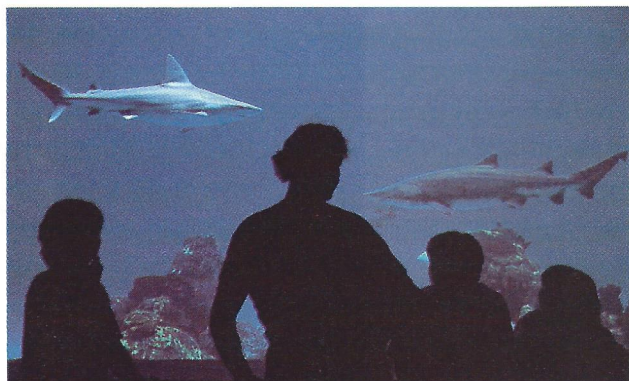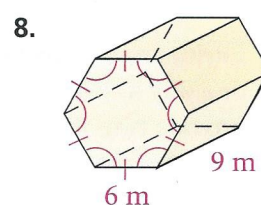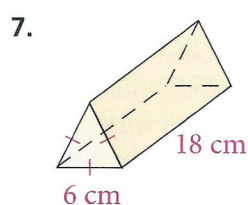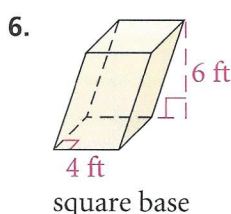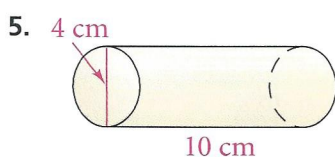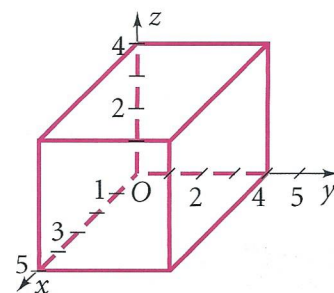
Choose Use mental math, paper and pencil, or a calculator to find the volume of each figure. When an answer is not an integer, round to the nearest tenth.
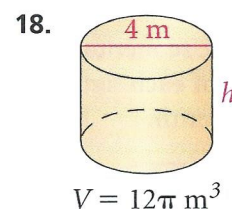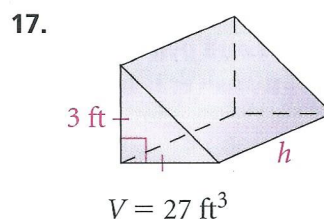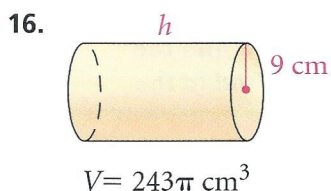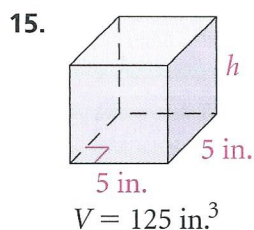
1.

8 in.

6 in.

2.

6 ft

6 ft

6 ft

3.

5 in.

8 in.

2 in.

4.

16 m

8 m
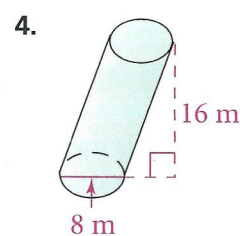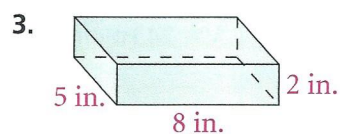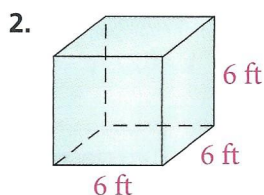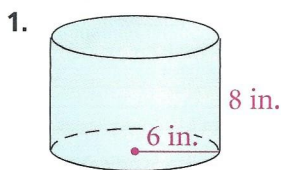
Calculator Find the volume of each figure. When an answer is not a whole number, round to the nearest tenth.

5.

4 cm

10 cm

6.

6 ft

4 ft

square base

7.

18 cm

6 cm

8.

9 m

6 m

9. a. What is the volume of a 7 ft-by-4 ft-by-1 ft waterbed mattress?
   b. To the nearest pound, what is the weight of the water in a full mattress? (Water weighs 62.4 lb/ft^3.)

10. Geometry in 3 Dimensions Find the volume of the rectangular prism at the right.

11. Open-ended Give the dimensions of two rectangular prisms that each have a volume of 80 cm^3 but have different surface areas.

12. Landscaping Zia is planning to landscape her backyard. The yard is a 70 ft-by-60 ft rectangle. She plans to put down a 4-in. layer of topsoil. She can buy bags of topsoil at $2.50 per 3-ft^3 bag, with free delivery. Or, she can buy bulk topsoil for $25.00 per yd^3, plus a $20 delivery fee. Which option is less expensive? Explain.

13. Water Resources One of the West Delaware water-supply tunnels is a 105-mi long cylinder with a diameter of 13.5 ft. To the nearest million cubic feet, how much earth was removed when the tunnel was built?

14. Standardized Test Prep The volume of a cube is 1000 cm^3. What is its surface area?
   A. 60 cm^2
   B. 600 cm^2
   C. 100 cm^2
   D. about 4630 cm^2
   E. cannot be determined

Find the height of each prism or cylinder with the given volume.

15.

$V = 125$ in.^{3}

5 in.
5 in.
h

16.

h

9 cm

$V = 243\pi$ cm^3

17.

3 ft

h

$V = 27$ ft^3

18.

4 m

h

$V = 12\pi$ m^3

19. a. What is the volume of the "ordinary" cube in the cartoon if each edge is 18 in. long?
  b. What is the volume of the "improved" cube if each edge is half as long as an edge of the "ordinary" cube?
  c. **Writing** Do you agree with the cartoon statement that the "improved" cube is half the size of the "ordinary" cube? Explain.

20. Environmental Engineering A scientist has suggested that one way to keep indoor air relatively pollution free is to provide two or three pots of flowers such as daisies for every 100 ft^2 of floor space with an 8-ft ceiling. How many pots of daisies would a 35 ft-by-45 ft-by-8 ft classroom need?

21. a. The volume of a cylinder is 600π cm^3. The radius of a base of the cylinder is 5 cm. What is the height of the cylinder?
  b. The volume of a cylinder is 135π cm^3. The height of the cylinder is 15 cm. What is the radius of a base of the cylinder?
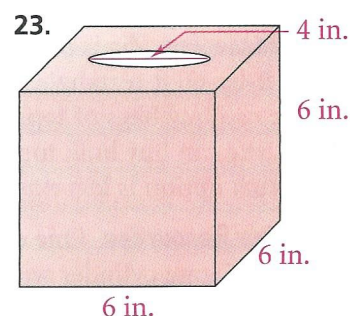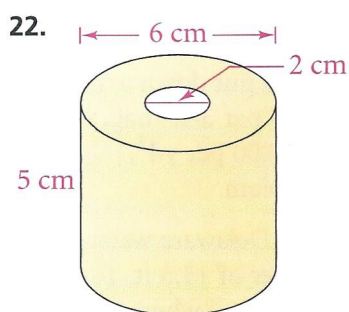
GUINDON © News America Syndicate, 1985
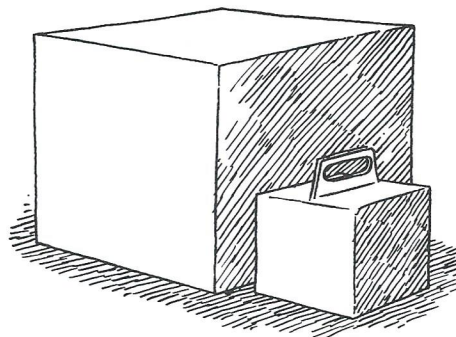
An improved cube (right), half the size of ordinary cubes. It has a convenient carrying handle.
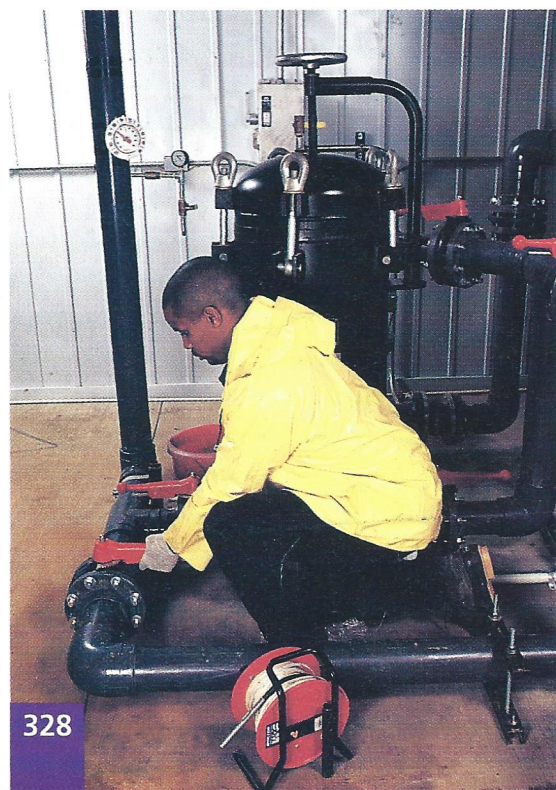
A cylinder has been cut out of each figure. Find the volume of the remaining figure. Round your answer to the nearest tenth.

22.

← 6 cm →

2 cm

5 cm

23.

4 in.

6 in.

6 in.

6 in.

6 in.

24. Plumbing The outside diameter of a pipe is 5 cm. The inside diameter is 4 cm. If the pipe is 4 m long, what is the volume of the metal used for this length of pipe? Round your answer to the nearest whole number.