1-5 Good Definitions

What You'll Learn

- Identifying a good definition
- Understanding the meaning of terms like *bisector* and *perpendicular*

...And Why

To sharpen a skill that is a key to communicating

WORK TOGETHER

Work with a partner to identify what makes a figure a *polyglob*.

1. Which figures above are polyglobs?
2. Describe what a polyglob is.

THINK AND DISCUSS

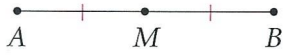
Properties of Good Definitions

In geometry we start with undefined terms such as *point, line,* and *plane* whose meanings we understand intuitively. Then we use those terms to define other terms such as *collinear points.*

A good definition can help you identify or classify an object. A good definition has several important components.

- ✔ A good definition uses clearly understood terms. The terms should be commonly understood or previously defined.
- ✔ A good definition is precise. Good definitions avoid words such as *large, sort of,* and *some.*
- ✔ A good definition states what the term *is,* rather than what it is not. A poor definition of *big* is "Big is the opposite of small."

A **midpoint** of a segment is a point that divides a segment into two congruent segments.

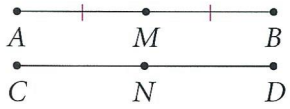


3. What previously defined terms are used in the definition of *midpoint?*

A good definition is *reversible.*

If $\overline{AM} \cong \overline{MB}$, then M is the midpoint of $\overline{AB}$.

If N is the midpoint of $\overline{CD}$, then $\overline{CN} \cong \overline{ND}$.



QUICK REVIEW

A right angle is an angle whose measure is 90.

4. a. If $\angle A$ is a right angle, what is the measure of $\angle A$?
b. If $m\angle B = 90$, classify $\angle B$.

Notice that you can use the definition of a right angle to justify your answer to each part of question 4.

One way to test a definition is to look for a *counterexample* that shows that the definition is wrong.

Example 1 **Relating to the Real World**

Language Arts Is the following an acceptable definition? Explain.

An airplane is a vehicle that flies.

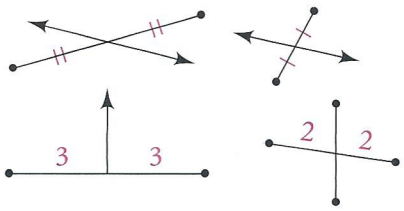
The definition is not acceptable because a helicopter is also a vehicle that flies, and a helicopter is not an airplane.

5. Try This Is the following an acceptable definition? Explain.

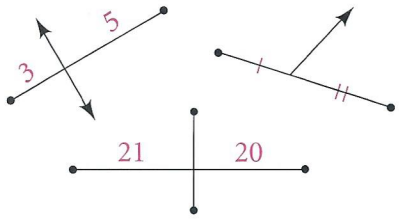
A square is a figure with four right angles.

Bisectors

6. Study the diagrams below and write a definition of a *segment bisector.*

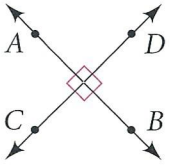


Segment bisectors



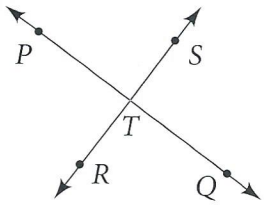
Not segment bisectors

Perpendicular lines are two lines that intersect to form right angles. The symbol $\perp$ is read as "is perpendicular to." In the diagram at the right, $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$.

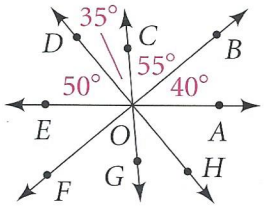


A $5 bill was folded in half. *How does the fold line meet the top step of the Lincoln Memorial?*

7. a. If $\overleftrightarrow{PQ} \perp \overleftrightarrow{RS}$, what is $m\angle PTR$? Explain.

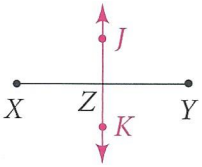


b. Which lines, if any, are perpendicular? Explain.



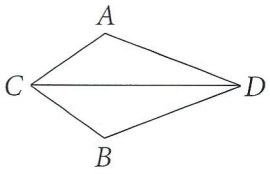
A **perpendicular bisector** of a segment is a line, segment, or ray that is perpendicular to a segment at its midpoint.

8. If you know that $\overleftrightarrow{JK}$ is the perpendicular bisector of $\overline{XY}$, what can you conclude about angles and segments in the diagram?

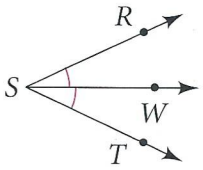


An **angle bisector** is a ray that divides an angle into two congruent angles.

9. a. Given: $\overrightarrow{CD}$ bisects $\angle ACB$. Name the congruent angles.

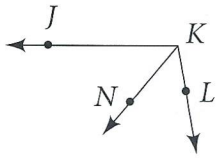


b. Given: $\angle RSW \cong \angle WST$ What can you conclude?

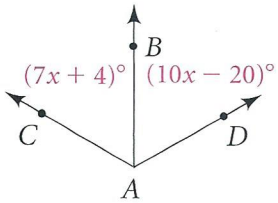


Example 2

$\overrightarrow{KN}$ bisects $\angle JKL$.
$m\angle JKN = 5x - 25$
$m\angle NKL = 3x + 5$
Solve for x and find $m\angle JKN$.



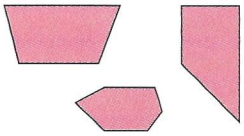
$\angle JKN \cong \angle NKL$, or $m\angle JKN = m\angle NKL$	Def. of $\angle$ bisector
$5x - 25 = 3x + 5$	Substitution
$5x - 25 + 25 = 3x + 5 + 25$	Add 25 to each side.
$5x = 3x + 30$	Simplify.
$5x - 3x = 3x - 3x + 30$	Subtract $3x$ from each side.
$2x = 30$	Simplify.
$x = 15$	Divide each side by 2.
$m\angle JKN = 5(15) - 25 = 50$	Substitute 15 for x.



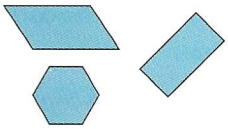
10. Try This In the diagram at the left, $\overrightarrow{AB}$ bisects $\angle CAD$. Solve for x and find $m\angle CAD$.

Exercises ON YOUR OWN

1. Which figures in the third group are *monopars*?

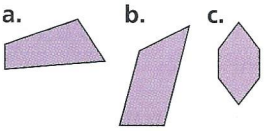


Monopars



Not monopars

a. b. c.



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Complete.

2. $DM = 8$, $MC =$ ■

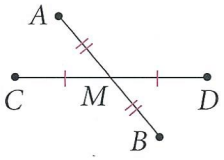
3. $MB = 6$, $AM =$ ■

4. $MC = 9$, $DC =$ ■

5. $AB = 10$, $AM =$ ■

6. $2MA =$ ■

7. $\frac{1}{2}DC =$ ■



8. ■ is the midpoint of ■ and ■.

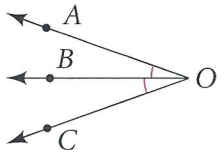
9. $m\angle AOB = 20$, $m\angle BOC =$ ■, $m\angle AOC =$ ■

10. $m\angle COA = 50$, $m\angle AOB =$ ■

11. $2m\angle AOB = m\angle$■

12. $\frac{1}{2}m\angle AOC = m\angle$■

13. ■ is the angle bisector of ■.



14. **Language Arts** Is the following an acceptable definition? Explain.

A cat is an animal with whiskers.

Write *true* or *false*.

15. $\overline{AM} \cong \overline{MB}$

16. M is the midpoint of $\overline{AB}$.

17. $\angle AMC \cong \angle CMB$

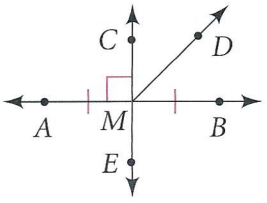
18. $\overline{CM} \perp \overline{AB}$

19. $\overleftrightarrow{MC} \perp \overleftrightarrow{MD}$

20. $m\angle CMB = 90$

21. M is the midpoint of $\overleftrightarrow{CE}$.

22. $\frac{1}{2}AB = AM$



Draw a figure for each description.

23. $\overleftrightarrow{AB} \perp \overleftrightarrow{BD}$

24. $\overline{AY}$ bisects $\overline{CX}$ at point Q.

25. $\overrightarrow{BQ}$ is the bisector of $\angle RBT$.

26. $\overrightarrow{AC}$ bisects right $\angle DAF$.

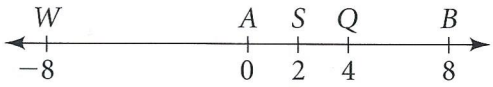
27. $\overline{AB}$ and $\overline{CT}$ are perpendicular bisectors of each other.

28. $\overline{RS}$ is the perpendicular bisector of $\overline{XY}$, but $\overline{XY}$ is not the perpendicular bisector of $\overline{RS}$.

29. What is the midpoint of $\overline{AB}$?

30. What is the coordinate of the midpoint of $\overline{QB}$?

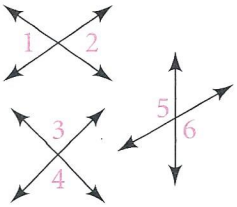
31. What is the coordinate of the midpoint of $\overline{WA}$?



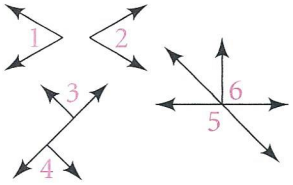
32. The coordinate of the midpoint of $\overline{AR}$ is -5. What is the coordinate of point R?

33. The coordinate of the midpoint of $\overline{ST}$ is 7. What is the coordinate of point T?

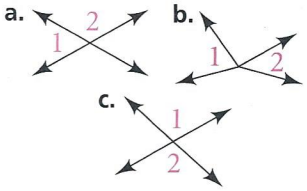
34. Which figures in the third group are *vertical angles?*



Vertical angles



Not vertical angles



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Algebra **Solve for x.**

35. $AB = 24, MB = 2x + 4$

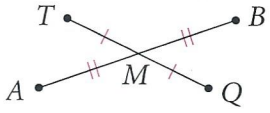
36. $TM = 3x + 5, MQ = x + 17$

37. $TQ = 4x + 16, TM = 20$

38. $MB = 8x + 7, AB = 126$

39. $TM = \frac{1}{2}x - 4, TQ = 12$

40. $AM = 5x - 1, AB = 38$



41. **a.** How many midpoints does a given segment have? How many bisectors does a given segment have?

b. Given a segment, consider any one plane that contains the segment. How many lines in that plane are perpendicular bisectors of the given segment?

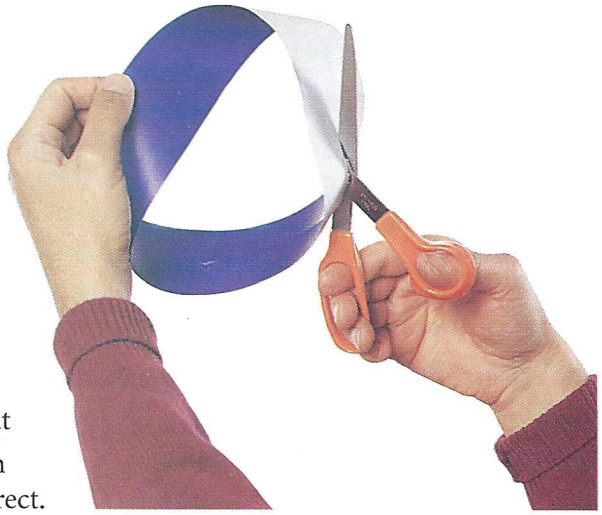
c. **Geometry in 3 Dimensions** Given a segment, how many lines are there in space that are perpendicular bisectors of the given segment?

42. **Coordinate Geometry** Find the coordinates of the midpoint of $\overline{AB}$ with endpoints $A(0, 5)$ and $B(0, 13)$.

43. **a.** **Manipulatives** Cut out a strip of paper about 11 in. long and 1 in. wide. Twist it once and tape the ends together. You now have a *Möbius band.*

b. Cut the Möbius band along its center. Does cutting the band bisect it? Explain your answer.

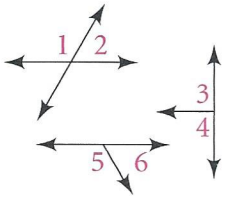
44. **Writing** If point M is the midpoint of $\overline{AB}$, you know that $\overline{AM} \cong \overline{MB}$. How is AM related to AB? Write an equation about AM and AB and explain why your equation is correct.



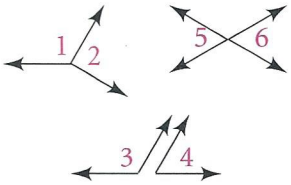
45. **Open-ended** Describe some perpendicular lines in your home or classroom.

46. **Language Arts** Is the following an acceptable definition? Explain.
An obtuse angle is an angle whose measure is greater than 90.

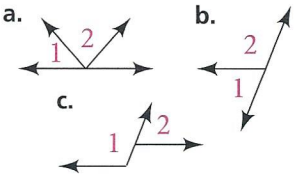
47. Which angles in the third group form a *linear pair?*



Linear pairs



Not linear pairs



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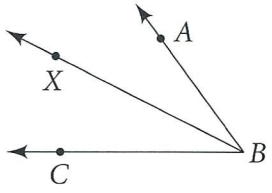
Algebra $\overrightarrow{BX}$ **is the bisector of** $\angle ABC$. **Complete each equation.**



48. $m\angle ABX = 5x$, $m\angle XBC = 3x + 10$, $m\angle ABC =$ ■

49. $m\angle ABC = 4x - 12$, $m\angle ABX = 24$, $x =$ ■

50. $m\angle ABX = 4x - 16$, $m\angle CBX = 2x + 6$, $x =$ ■

51. $m\angle ABC = 5x + 18$, $m\angle CBX = 2x + 12$, $m\angle ABC =$ ■

52. **Critical Thinking** Lee knows that whenever $\angle ABC$ has $\overrightarrow{BX}$ as an angle bisector, $\angle ABX \cong \angle CBX$. Lee claims there is always a related equation, $m\angle ABX = \frac{1}{2}m\angle ABC$. Her friend Clarissa claims the related equation is $2m\angle ABX = m\angle ABC$. Which equation is correct? Explain.
A diagram may be helpful.

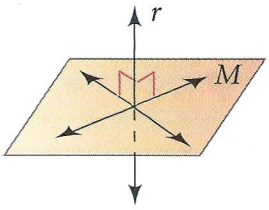
53. **Standardized Test Prep** Point M is the midpoint of $\overline{PQ}$. Which of these is *not* true?

A. $\overline{PM} \cong \overline{MQ}$ **B.** $PM + MQ = PQ$ **C.** $MQ = \frac{1}{2}PQ$

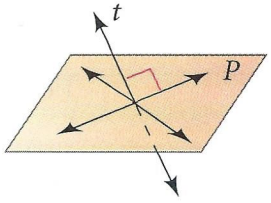
D. $\overrightarrow{PM}$ and $\overrightarrow{PQ}$ are opposite rays. **E.** $PQ = 2PM$

54. **Writing** Write a definition of a line parallel to a plane.

55. Study the figures below. Complete the definition of a line perpendicular to a plane. A line is perpendicular to a plane if it is _?_ to every line in the plane that _?_.



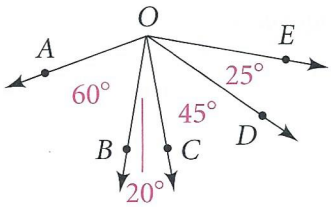
Line $r \perp$ plane M.



Line t is not $\perp$ plane P.

Exercises MIXED REVIEW

Use the diagram at the right for Exercises 56–60.



56. Find each measure.

a. $m\angle AOC$ **b.** $m\angle AOD$

c. $m\angle DOB$ **d.** $m\angle BOE$

57. Name an obtuse angle. **58.** Name an acute angle.

59. Name a right angle. **60.** Name all the rays.

61. Draw opposite rays $\overrightarrow{RS}$ and $\overrightarrow{RW}$. Find RS if $SW = 8$ and $RW = 5$.

Getting Ready For Lesson 1-6

62. $\overleftrightarrow{AX}$ is the perpendicular bisector of $\overline{QS}$ at point M. Name two congruent segments.

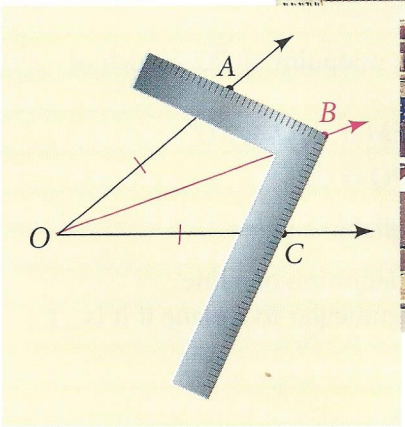
63. $\overrightarrow{PT}$ is the bisector of $\angle APR$. Name two congruent angles.

64. $\overrightarrow{OR}$ is the bisector of right $\angle TOS$. Find $m\angle TOR$.

Geometry at Work

Cabinetmaker

Cabinetmakers make not only cabinets but all types of wooden furniture. The artistry of cabinetmaking can be seen in the beauty and uniqueness of the finest doors, shelves, and tables. The craft of the profession is in knowing which types of wood and tools to use, and how to use them.



The carpenter's square is one of the most useful of the cabinetmaker's tools. It can be applied to a variety of measuring tasks. The figure shows how to use a carpenter's square to bisect $\angle O$.

First, mark equal lengths OA and OC on the sides of the angle. Then position the square so that $AB = BC$ to locate point B. Finally, draw $\overrightarrow{OB}$. $\overrightarrow{OB}$ bisects $\angle O$.

Mini Project: Make a carpenter's square out of cardboard. Mark the edges in equal intervals as shown in the figure. Draw a line segment. Then demonstrate how you can use the square to draw the perpendicular bisector of the segment.