6-3 Surface Areas of Pyramids and Cones

What You'll Learn

- Finding the lateral areas and surface areas of pyramids and cones

...And Why

To find the lateral areas of pyramids, such as the Great Pyramid at Giza, and of conical tower roofs

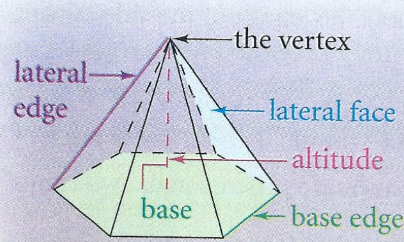
What You'll Need

calculator, metric ruler, compass, protractor, scissors, tape

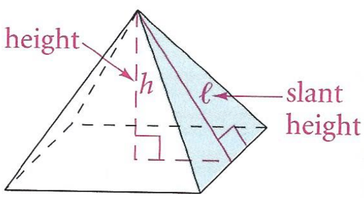
THINK AND DISCUSS

Lateral Areas and Surface Areas of Pyramids

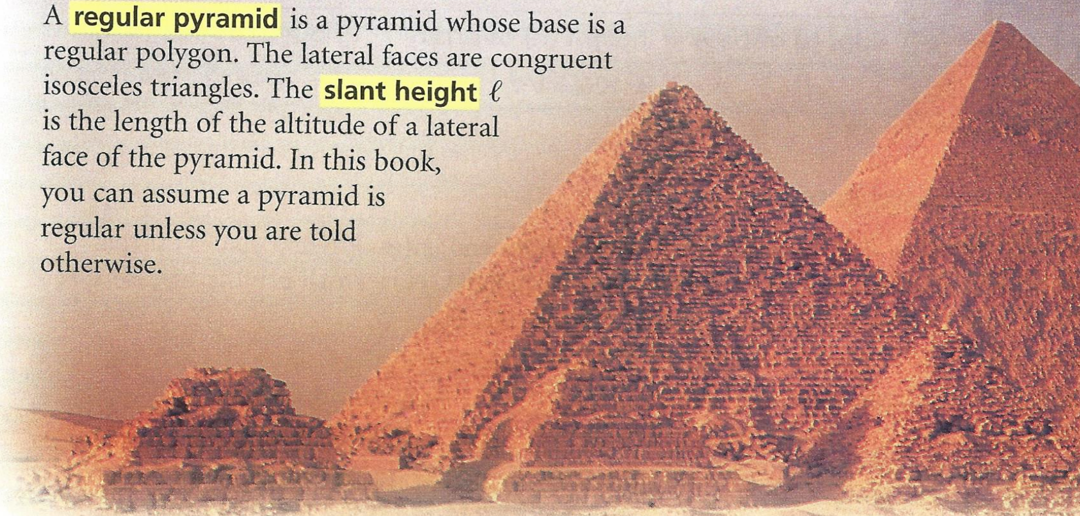
Many Egyptian pharaohs built pyramids as burial tombs. The Fourth Dynasty, 2615–2494 B.C., was the age of the great pyramids. The builders knew and understood the mathematical properties of a pyramid.

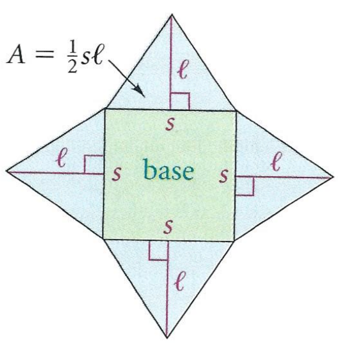


A **pyramid** is a polyhedron in which one face (the **base**) can be any polygon and the other faces (the **lateral faces**) are triangles that meet at a common vertex (called the **vertex** of the pyramid). You can name a pyramid by the shape of its base. The **altitude** of a pyramid is the perpendicular segment from the vertex to the plane of the base. The length of the altitude is the **height** h of the pyramid.



A **regular pyramid** is a pyramid whose base is a regular polygon. The lateral faces are congruent isosceles triangles. The **slant height** ℓ is the length of the altitude of a lateral face of the pyramid. In this book, you can assume a pyramid is regular unless you are told otherwise.





You can find a formula for the lateral area of a pyramid by looking at its net. The **lateral area** is the sum of the areas of the congruent lateral faces.

$$\text{L.A.} = 4(\tfrac{1}{2}s\ell) \quad \text{The area of each lateral face is } \tfrac{1}{2}s\ell.$$
$$= \tfrac{1}{2}(4s)\ell \quad \text{Commutative Property of Multiplication}$$
$$= \tfrac{1}{2}p\ell \quad \text{The perimeter } p \text{ of the base is } 4s.$$

To find the **surface area** of a pyramid, add the area of its base to its lateral area.

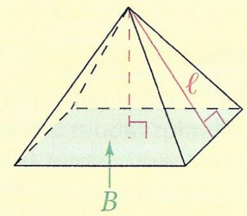
Theorem 6-3
Lateral and Surface Areas of a Regular Pyramid

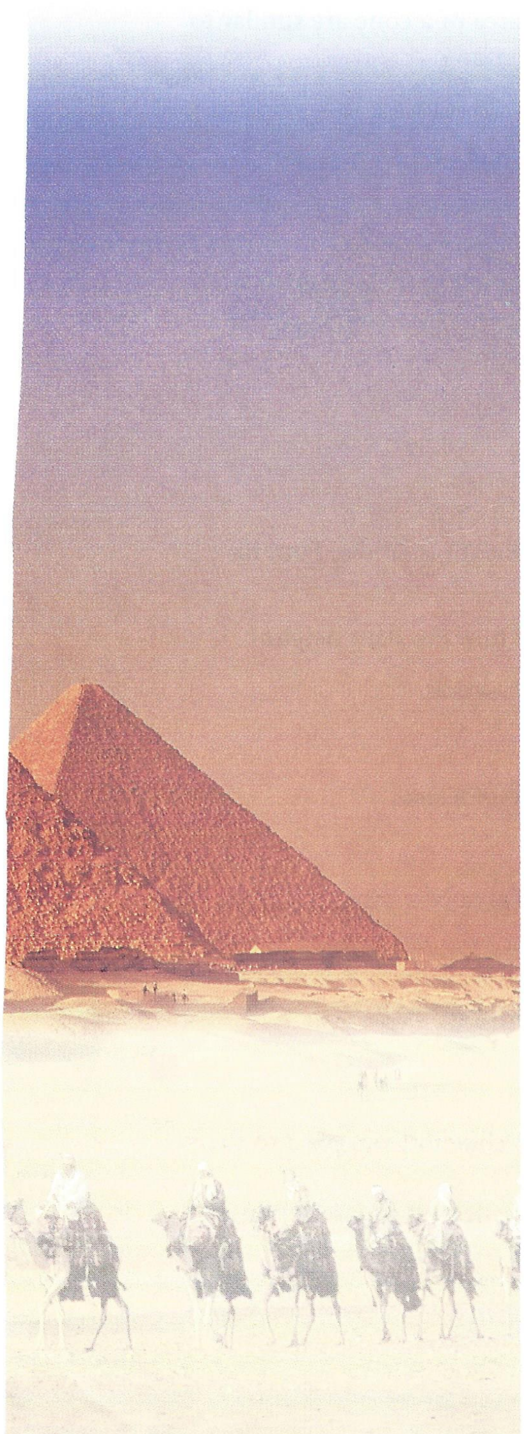
The lateral area of a regular pyramid is half the product of the perimeter of the base and the slant height.

$$\text{L.A.} = \frac{1}{2}p\ell$$

The surface area of a regular pyramid is the sum of the lateral area and the area of the base.

$$\text{S.A.} = \text{L.A.} + B$$





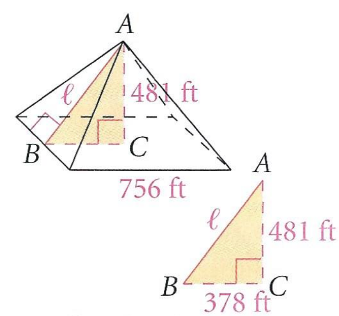
1. **Try This** A regular hexagonal pyramid has base edges 60 m long and slant height 35 m. Find the perimeter of its base and its lateral area.

Sometimes the slant height of a pyramid is not given. You must calculate it before you can find the lateral or surface area.

Example 1 **Relating to the Real World**

Social Studies The Great Pyramid at Giza, Egypt, was built about 2580 B.C. as a final resting place for Pharaoh Khufu. At the time it was built, its height was about 481 ft. Each edge of the square base was about 756 ft long. What was the lateral area of the pyramid?

- The legs of right $\triangle ABC$ are the height of the pyramid and the apothem of the base. The height of the pyramid is 481 ft. The apothem of the base is $\frac{756}{2}$, or 378 ft. You can use the Pythagorean Theorem to find the slant height ℓ.



$\ell^2 = AC^2 + BC^2$

$\ell^2 = 481^2 + 378^2$

$\ell = \sqrt{481^2 + 378^2}$ Find the square root of each side.

481 [x^2] [+] 378 [x^2] [=] [$\sqrt{\ }$] 611.75567

- Now use the formula for the lateral area of a pyramid. The perimeter of the base is approximately $4 \cdot 756$, or 3024 ft.

$\text{L.A.} = \frac{1}{2}p\ell$

$= \frac{1}{2}(3024)(611.75567)$ Substitute.

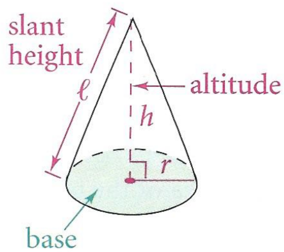
0.5 [×] 3024 [×] 611.75567 [=] 924974.57

The lateral area of the Great Pyramid at Giza was about 925,000 ft^2.

2. **Try This** Find the surface area of the Great Pyramid at Giza.

> **Who?** The Greek mathematician Apollonius (about 225 B.C.) wrote a work about the cone and its cross sections. His *Conic Sections* earned him the title "The Great Geometer."

Lateral Areas and Surface Areas of Cones



A **cone** is like a pyramid, but its base is a circle. In a **right cone** the **altitude** is a perpendicular segment from the vertex to the center of the base. The **height** h is the length of the altitude. The **slant height** ℓ is the distance from the vertex to a point on the edge of the base. In this book, all the cones discussed will be right cones.

The formulas for the lateral area and surface area of a cone are similar to those for a pyramid.

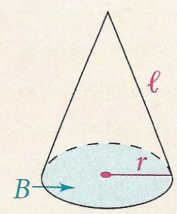
Theorem 6-4
Lateral and Surface Areas of a Right Cone

The lateral area of a right cone is half the product of the circumference of the base and the slant height.

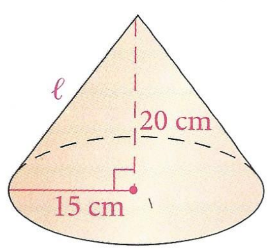
$$\text{L.A.} = \frac{1}{2} \cdot 2\pi r \cdot \ell, \text{ or L.A.} = \pi r \ell$$

The surface area of a right cone is the sum of the lateral area and the area of a base.

$$\text{S.A.} = \text{L.A.} + B$$



Example 2



The radius of the base of a cone is 15 cm. Its height is 20 cm. Find its lateral area in terms of π.

- To determine the lateral area, you first must find the slant height ℓ.

$\ell^2 = 20^2 + 15^2$	**Use the Pythagorean Theorem.**
$= 400 + 225$	**Simplify.**
$= 625$	
$\ell = 25$	**Find the square root of each side.**

The slant height of the cone is 25 cm.

- Now you can use the formula for the lateral area of a cone.

$\text{L.A.} = \pi r \ell$	
$= \pi(15)25$	**Substitute.**
$= 375\pi$	**Simplify.**

The lateral area of the cone is $375\pi\text{ cm}^2$.

3. a. **Calculator** Find the lateral area of the cone in Example 2 to the nearest square centimeter.

b. Find the surface area of the cone to the nearest square centimeter.

WORK TOGETHER

- Work in a group. Use a compass to draw three congruent circles with radii 8 cm. Use a protractor to draw the shaded sectors shown below.

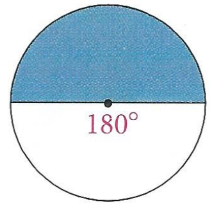


Figure 1

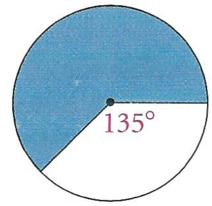


Figure 2

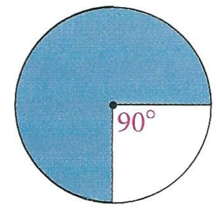


Figure 3

- Cut out the sectors. Tape the radii of each sector together, without overlapping, to form a cone without a base.

4. Find the slant height of each cone.

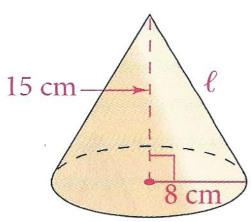
5. Find the circumference *C* and radius *r* of each cone. Then find the lateral area of each cone. Record your results in a table like the one shown at the left.

Cone	*C*	*r*	L.A.
1			
2			
3			

Exercises ON YOUR OWN

Choose **Use mental math, pencil and paper, or a calculator to find the slant height of each figure.**

1.



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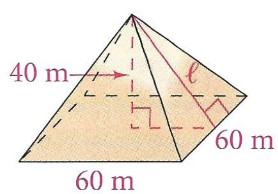

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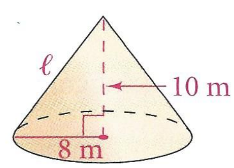

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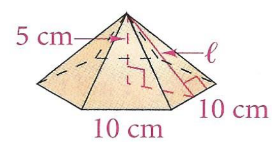
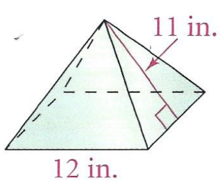


Find the lateral area of each figure. You may leave answers in terms of π.

5.



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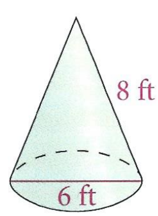

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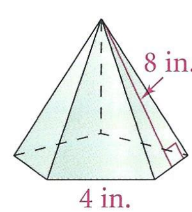

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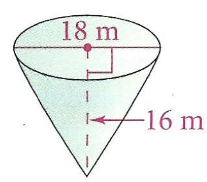
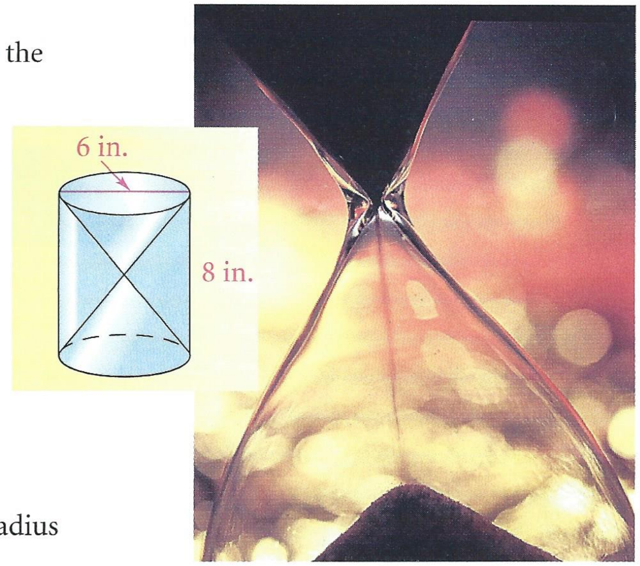


9. **Writing** How are a cone and a pyramid alike? How are they different?

10. **Architecture** The roof of the tower in a castle is shaped like a cone. The height of the roof is 30 ft and the radius of the base is 15 ft. What is the area of the roof? Round your answer to the nearest tenth.

11. **Critical Thinking** Anita says that when she uses her calculator to find the surface area of a cone she uses the formula S.A. $= (\ell + r)r\pi$. Explain why this formula works. Why do you think Anita uses this formula?

12. **Manufacturing** The hourglass shown at the right is made by connecting two glass cones inside a glass cylinder. Which has more glass, the two cones or the cylinder? Explain.



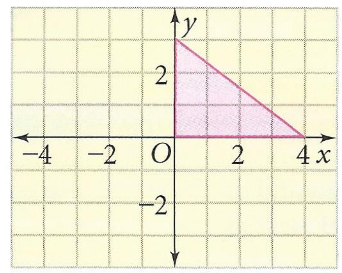
13. A regular square pyramid has base edges 10 in. long and height 4 in. Sketch the pyramid and find its surface area. Round your answer to the nearest tenth.

14. **Algebra** The lateral area of a cone is 48π in.^{2}. The radius is 12 in. Find the slant height.

15. **Open-ended** Draw a pyramid with a lateral area of 48 cm^2. Label its dimensions. Then find its surface area.

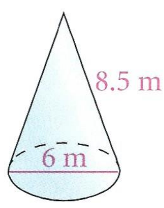
16. a. **Coordinate Geometry** Describe the figure that is formed when the right triangle at the right is rotated 360° about the x-axis. Find its lateral area. Leave your answer in terms of π.

    b. What is the lateral area of the figure formed when the triangle is rotated 360° about the y-axis? Leave your answer in terms of π.



Calculator **Find the lateral area of each figure to the nearest tenth.**

17.



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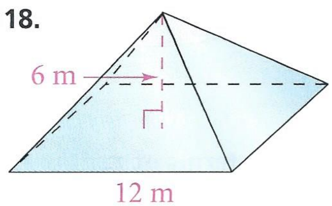

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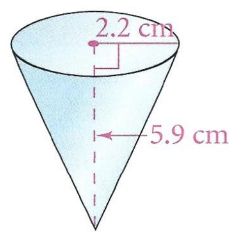

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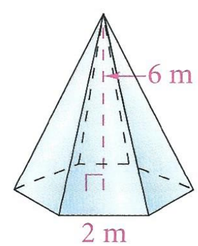
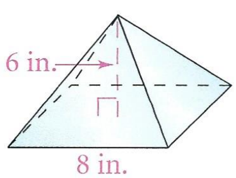


Calculator **Find the surface area of each figure to the nearest tenth.**

21.



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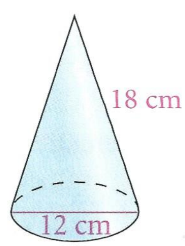

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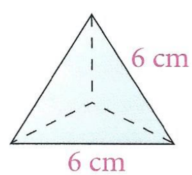

24.