

Circles: Circumference and Arc Length

What You'll Learn

- Finding the circumference of a circle and the length of an arc

...And Why

To solve problems involving auto safety, metalworking, and amusement park rides

What You'll Need

- circular objects
- string
- metric ruler
- calculator

WORK TOGETHER

Work in groups. Each member of your group should have one circular object such as a juice can or a jar lid.

- Measure the diameter of each circle to the nearest millimeter.
- Find the circumference of each circle by wrapping a string around each object. Straighten the string and measure its length to the nearest tenth of a centimeter.
- Organize your group's data in a table like the one below. Calculate the ratio $\frac{\text{circumference}}{\text{diameter}}$ to the nearest hundredth.

Name of Object	Circumference (C)	Diameter (d)	$\frac{C}{d}$
jelly-jar lid	19.6 cm	6.2 cm	3.16

- Make a **conjecture** about the relationship between the circumference and the diameter of a circle.

THINK AND DISCUSS

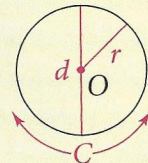
Circumference

The ratios you found in the Work Together are estimates of the number **pi** (π), the ratio of the circumference of a circle to its diameter.

Theorem 5-10 Circumference of a Circle

The circumference of a circle is π times the diameter.

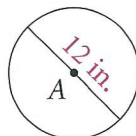
$$C = \pi d \text{ or } C = 2\pi r$$



Example 1

Find the circumference of $\odot A$ and $\odot B$. Leave your answer in terms of π .

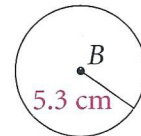
a.



$$C = \pi d$$

$$C = 12\pi \text{ in.}$$

b.



$$C = 2\pi r$$

$$C = 2 \cdot \pi \cdot 5.3$$

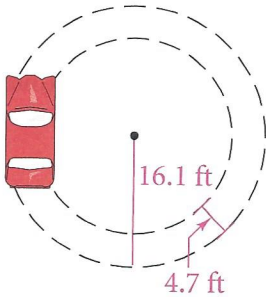
$$C = 10.6\pi \text{ cm}$$

- Try This** What is the radius of a circle with circumference 18π m?

Since the number π is irrational, you cannot write it as a decimal. You can use 3.14, $\frac{22}{7}$, or the π key on your calculator as approximations for π .

Two circles that lie in the same plane and have the same center are **concentric circles**.

Example 2 Relating to the Real World



Automobiles A manufacturer advertises that a new car has a turning radius of only 16.1 ft. The distance between the two front tires is 4.7 ft. How much farther do the outside tires have to travel in making a complete circle than the tires on the inside?

The outside and inside tires travel on concentric circles. The radius of the outer circle is 16.1 ft. To find the radius of the inner circle, you must subtract 4.7 ft.

$$\text{circumference of outer circle} = 2\pi(16.1) = 32.2\pi$$

$$\text{radius of the inner circle} = 16.1 - 4.7 = 11.4 \text{ ft}$$

$$\text{circumference of inner circle} = 2\pi(11.4) = 22.8\pi$$

The difference in the two distances is $32.2\pi - 22.8\pi = 9.4\pi$.

$$9.4 \times \pi = 29.530971$$

The outside tires travel about 29.5 ft farther than the inside tires.

- 3. Try This** The diameter of a bicycle wheel is 26 in. To the nearest whole number, how many revolutions does the wheel make when the bicycle travels 100 yd?



Arc Length

In Chapter 2 you found the measure of an arc in degrees. You can also find the **arc length**, which is a fraction of a circle's circumference.

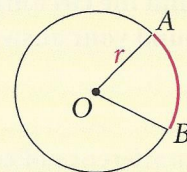
An arc of 60° represents $\frac{60}{360}$ or $\frac{1}{6}$ of the circle. Its arc length is $\frac{1}{6}$ the circumference of the circle.

This observation suggests the following generalization.

Theorem 5-11 Arc Length

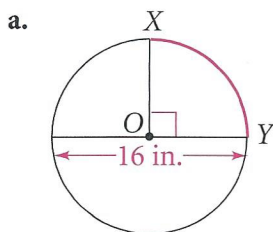
The length of an arc of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the circumference of the circle.

$$\text{length of } \widehat{AB} = \frac{m\widehat{AB}}{360} \cdot 2\pi r$$

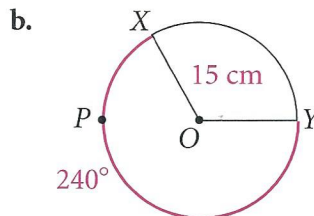


Example 3

Find the length of the arc shown in red on each circle. Leave your answer in terms of π .



$$\begin{aligned}\text{length of } \widehat{XY} &= \frac{m\widehat{XY}}{360} \cdot \pi d \\ \text{length of } \widehat{XY} &= \frac{90}{360} \cdot \pi(16) \\ &= 4\pi \text{ in.}\end{aligned}$$



$$\begin{aligned}\text{length of } \widehat{XPY} &= \frac{m\widehat{XPY}}{360} \cdot 2\pi r \\ \text{length of } \widehat{XPY} &= \frac{240}{360} \cdot 2\pi(15) \\ &= 20\pi \text{ cm}\end{aligned}$$

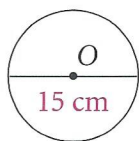
4. **Try This** Find the length of a semicircle with radius 1.3 m. Leave your answer in terms of π .
5. **Critical Thinking** Is it possible for two arcs of different circles to have the same measure, but different lengths? Support your answer with an example.
6. **Critical Thinking** Is it possible for two arcs of different circles to have the same length but have different measures? Support your answer with an example.

Your answers to Questions 5 and 6 illustrate that for arcs to be congruent two things must be true. **Congruent arcs** are arcs that have the same measure and are in the same circle or in congruent circles.

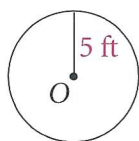
Exercises ON YOUR OWN

Find the circumference of $\odot O$. Leave your answer in terms of π .

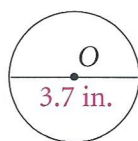
1.



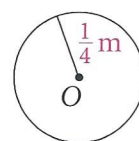
2.



3.



4.



Calculator Find the circumference of each circle with the given radius or diameter. Round your answer to the nearest hundredth.

5. $r = 9$ in.

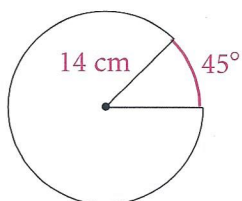
6. $d = 7.3$ m

7. $d = \frac{1}{2}$ yd

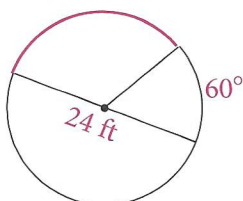
8. $r = 0.13$ cm

Find the circumference of each circle. Then find the length of the arc shown in red on each circle. Leave your answer in terms of π .

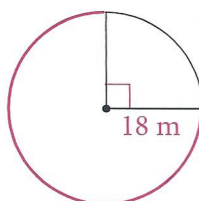
9.



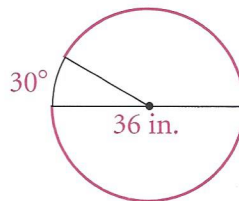
10.



11.



12.



13. The circumference of a circle is 100π in. Find each of the following.

a. the diameter

b. the radius

c. the length of an arc of 120°

14. **Coordinate Geometry** The endpoints of a diameter of a circle are $A(1, 3)$ and $B(4, 7)$. Find each of the following.

a. the coordinates of the center b. the diameter

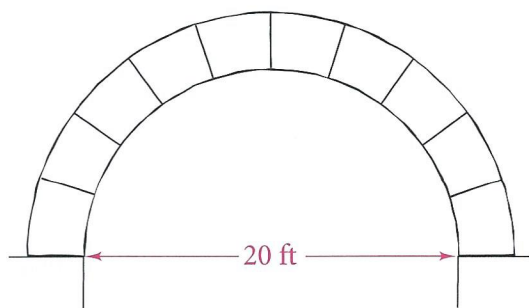
c. the circumference

15. **Metalworking** Miya constructed a wrought-iron arch to top the entrance to a mall. The 11 bars between the two concentric semicircles are each 3 ft long. Find the length of the wrought iron used to make this structure. Round your answer to the nearest foot.

16. A 60° arc of $\odot A$ has the same length as a 45° arc of $\odot B$. Find the ratio of the radius of $\odot A$ to the radius of $\odot B$.

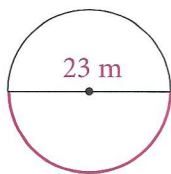
17. **Space Travel** The orbit of the space station *Mir* is 245 mi above Earth. How much greater is the circumference of *Mir*'s orbit than the circumference of Earth? Earth's radius is about 3960 mi. Leave your answer in terms of π .

18. **Open-ended** Use a compass and protractor to draw two noncongruent arcs with the same measure.

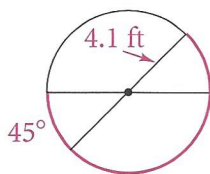


Calculator Find the length of the arc shown in red on each circle. Round your answer to the nearest hundredth.

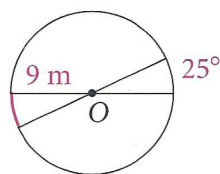
19.



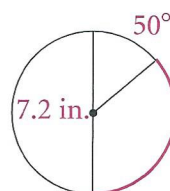
20.



21.



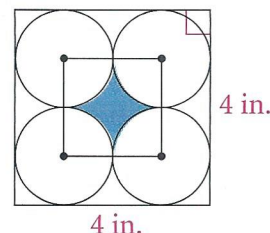
22.



23. Find the perimeter of the shaded portion of the figure at the right. Leave your answer in terms of π .

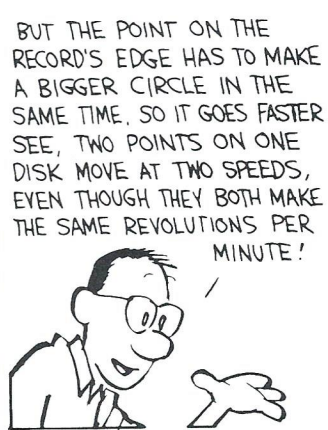
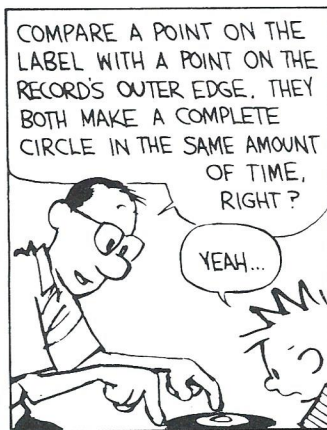
24. **Standardized Test Prep** The length of \widehat{AB} is 6π cm and $m\widehat{AB} = 120$. What is the diameter of the circle?

- A. 2 cm B. 6 cm C. 9 cm
D. 18 cm E. 24 cm



25. **Coordinate Geometry** Find the length of a semicircle with endpoints $(3, 7)$ and $(3, -1)$. Round your answer to the nearest tenth.

CALVIN AND HOBBS by Bill Watterson



Cartoon Use what you learned from Calvin's father to answer the following questions.

26. In one revolution, how much farther does a point 10 cm from the center of the record travel than a point 3 cm from the center? Round your answer to the nearest hundredth.
27. **Writing** Kendra and her mother plan to ride the merry-go-round. Two horses on the merry-go-round are side by side. For a more exciting ride, should Kendra sit on the inside or the outside? Explain your reasoning.

