5-5   Areas of Trapezoids

WORK TOGETHER

Work in groups. Fold a piece of lined paper in half along one of the lines. On two lines of the paper, draw parallel segments of different lengths. Connect the endpoints of the segments to form a trapezoid. Cut through both layers of the folded paper, so that you will have two congruent trapezoids. Label b_1, b_2, and h for each trapezoid.

- Arrange the congruent trapezoids to form a parallelogram as shown at the left below.

1. a. Write an expression for the length of the base of the parallelogram.
    b. Write an expression for the area of the parallelogram using b_1, b_2, and h.

2. How does the area of each trapezoid compare to the area of the parallelogram?

3. Use your answers to Questions 1 and 2 to write a formula for the area of each trapezoid.

THINK AND DISCUSS

In a trapezoid, the parallel sides are the **bases.** The nonparallel sides are the **legs.** The **height** h is the perpendicular distance between the two parallel bases.
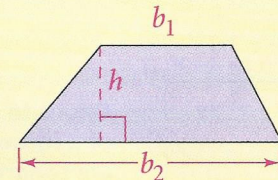
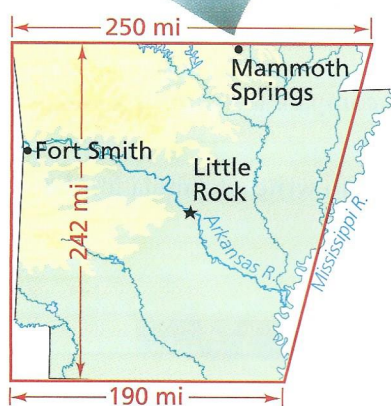Your observations in the Work Together suggest the following theorem.
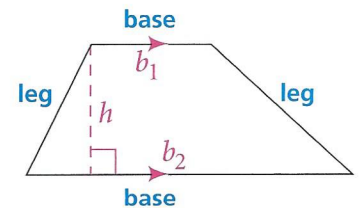
Theorem 5-8
Area of a Trapezoid

The area of a trapezoid is half the product of the height and the sum of the lengths of the bases.

$$A = \tfrac{1}{2}h(b_1 + b_2)$$

4. **Critical Thinking** When finding the area of a trapezoid, does it make a difference which base is labeled b_1 and which base is labeled b_2? Explain.

Example 1 Relating to the Real World

Geography Approximate the area of Arkansas by finding the area of the trapezoid shown.

$$A = \tfrac{1}{2}h(b_1 + b_2)$$ Use the area formula for a trapezoid.

$$= \tfrac{1}{2}(242)(190 + 250)$$ Substitute.

$$= 53{,}240$$

The area of Arkansas is about 53,240 mi².

5. Try This Find the area of a trapezoid with height 7 cm and bases 12 cm and 15 cm.

Sometimes properties of special right triangles can help you find the area of a trapezoid.

Example 2

Find the area of trapezoid *PQRS*. Leave your answer in simplest radical form.

You can draw an altitude that divides the trapezoid into a rectangle and a 30°-60°-90° triangle. Find *h*.

$$h = 2\sqrt{3}$$ longer leg = shorter leg · $\sqrt{3}$

$$A = \tfrac{1}{2}h(b_1 + b_2)$$ Use the area formula for a trapezoid.

$$= \tfrac{1}{2}(2\sqrt{3})(5 + 7)$$ Substitute.

$$= 12\sqrt{3}$$ Simplify.

The area of trapezoid *PQRS* is $12\sqrt{3}$ m².
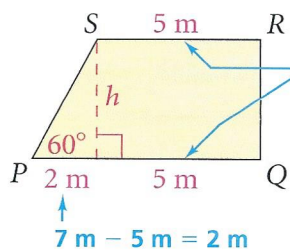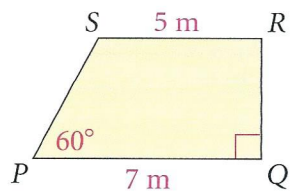
Opposite sides of a rectangle are congruent.

7 m − 5 m = 2 m

6. Suppose $m\angle P = 45$. Find the area of trapezoid *PQRS*.

Find the area of each trapezoid.

1.
21 in.
16 in.
38 in.

2.
24.3 cm
8.5 cm
9.7 cm

3.
5 ft
3 ft
6 ft

4.
6 m
10 m
8 m
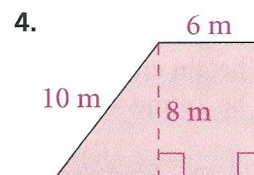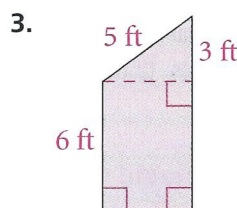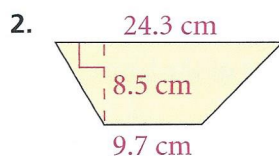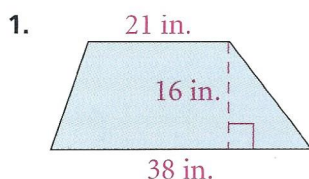
5. **Geography** Approximate the area of Nevada by finding the area of the trapezoid shown.

6. **Research** On a state map, select a town or county that is shaped like a trapezoid. Use the scale of the map to find values for b_1, b_2, and h. Then approximate the area.

7. The area of a trapezoid is 80 ft^2. Its bases have lengths 26 ft and 14 ft. Find its height.

8. **a.** A trapezoid has two right angles, bases of lengths 12 m and 18 m, and a height of 8 m. Sketch the trapezoid.
   b. What is the perimeter?
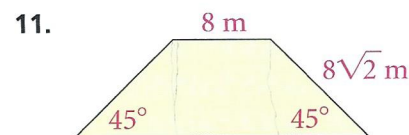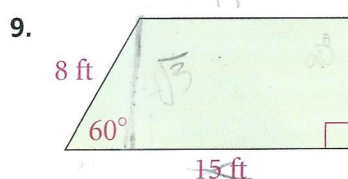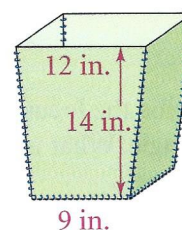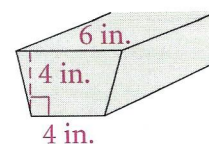   **c.** What is the area?

212 mi
Humboldt R.
315 mi
Reno
Carson City
480 mi
Las Vegas

Find the area of each trapezoid. Leave your answer in simplest radical form.

9.
8 ft
60°
15 ft

10.
13 in.
15 in.
9 in.
45°

11.
8 m
8√2 m
45°   45°
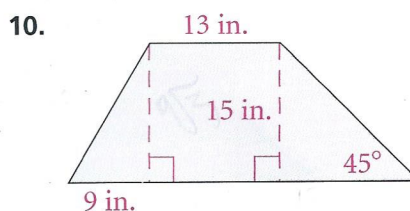
12. **Geometry in 3 Dimensions** A rain gutter has a trapezoidal cross section. The bottom is 4 in. wide, the top is 6 in. wide, and the gutter is 4 in. deep. What is the area of an end-piece?

6 in.
4 in.
4 in.

13. Draw a trapezoid. Label its bases and height b_1, b_2, and h. Then draw a diagonal of the trapezoid.
   a. Write an expression for the area of each triangle determined by the diagonal.
   b. **Writing** Explain how you can justify the trapezoid area formula using the areas of the two triangles.

14. **Open-ended** Draw a trapezoid. Measure its height and the lengths of its bases. Find its area.

15. **Crafts** You plan to lace together four isosceles trapezoids and a square to make the trash basket shown. How much material will you need?

12 in.
14 in.
9 in.

16. The area of an isosceles trapezoid is 160 cm². Its height is 8 cm and the length of its shorter base is 14 cm. Find the length of the longer base.

17. **Algebra** One base of a trapezoid is twice as long as the other. The height is the average of the two bases. The area is 324 cm². Find the height and the lengths of the bases. (*Hint:* Let the lengths of the bases be 2x and 4x.)

Calculator **Find the area of each trapezoid to the nearest tenth.**

18.

6.4 m
8.0 m
6.2 m   8.2 m

19.

8 ft
30°
|←9 ft→|

20.

1.7 m
45°
2.1 m
0.9 m

21.

4 cm
3 cm
3 cm   1 cm
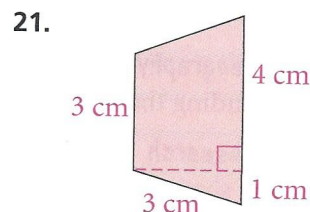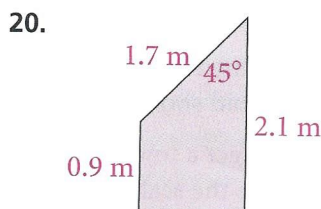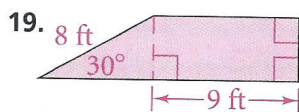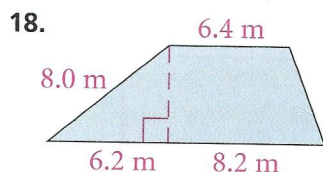
22. a. **Coordinate Geometry** Graph the lines $x = 0$, $x = 6$, $y = 0$, and $y = x + 4$.
   b. What quadrilateral do the lines form?
   c. Find the area of the quadrilateral.

23. **Recreation** A town youth center is building a skateboarding ramp. The ramp is 4 m wide, and the surface of the ramp is modeled by the equation $y = 0.25x^2$. You want to paint the front face of the ramp. Use the triangles and trapezoids shown to approximate the area of the face.

$y = 0.25x^2$

−2   −1   O   1   2   x
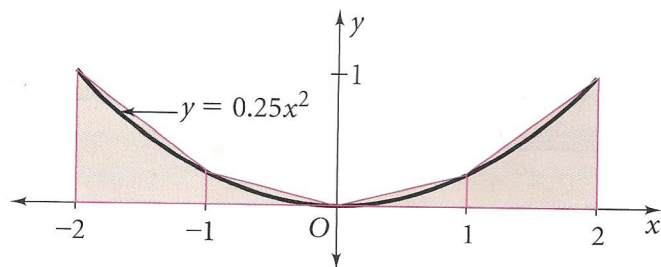
Exercises   **MIXED REVIEW**

Open-ended Find a possible length for the third side of a triangle that has two sides with the given lengths.

24. 7 cm, 10 cm   **25.** 2 in., 8 in.   **26.** 13 mm, 6 mm   **27.** 4 ft, 9 ft

28. **Locus** Describe the locus of points in a plane equidistant from the sides of an angle. What is another name for this locus?