5-4 Special Right Triangles

What You'll Learn

- Using the properties of 45°-45°-90° and 30°-60°-90° triangles

...And Why

To study figures in real life, including baseball diamonds and helicopter blades, which use special right triangles

What You'll Need

- centimeter grid paper
- metric ruler
- calculator
- protractor

WORK TOGETHER

Work in a group. Have each person draw a different isosceles right triangle on centimeter grid paper. Choose integer values for the lengths of the legs.

- Record the length of each leg. Then use the Pythagorean Theorem to find the length of the hypotenuse. Leave your answers in simplest radical form.
- Organize your group's data in a table like the one below. Look for a pattern relating the side lengths of each triangle.

Triangle	Leg Length	Hypotenuse Length
Triangle 1		
Triangle 2		

- Make a **conjecture** about the relationship between the lengths of the legs and the length of the hypotenuse of an isosceles right triangle.

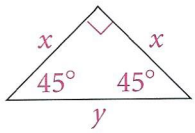
THINK AND DISCUSS

45°-45°-90° Triangles

1. What do you know about the measures of the acute angles of an isosceles right triangle?
2. If the measures of the angles of a triangle are 45, 45, and 90, why are the legs of the triangle congruent?

Another name for an isosceles right triangle is a 45°-45°-90° triangle.

3. **a.** Use the Pythagorean Theorem to solve for y in terms of x. Leave your answer in simplest radical form.
   b. Do the results of part (a) agree with the pattern you found in the Work Together?



4. Find the value of each variable *without* using the Pythagorean Theorem.

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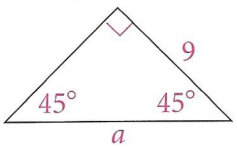

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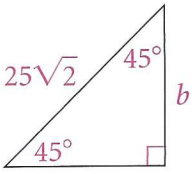

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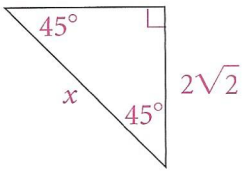
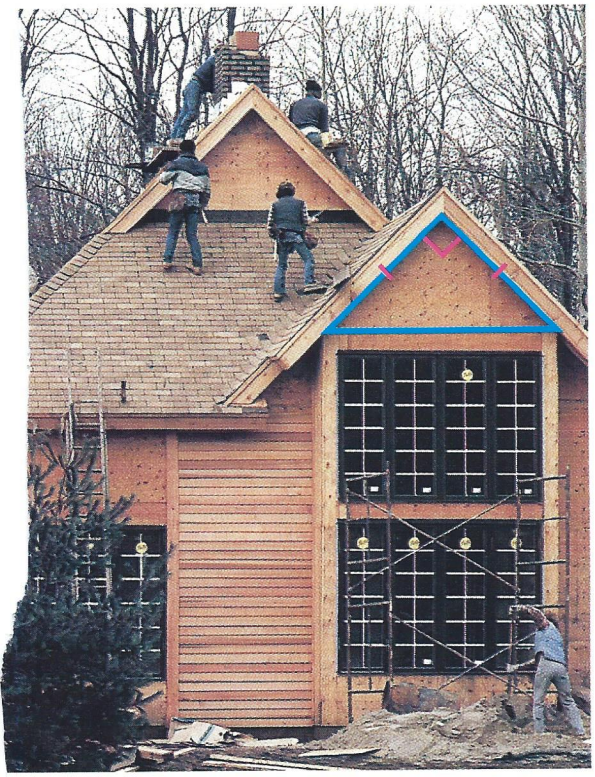


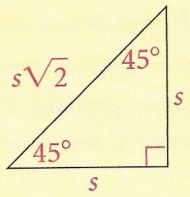


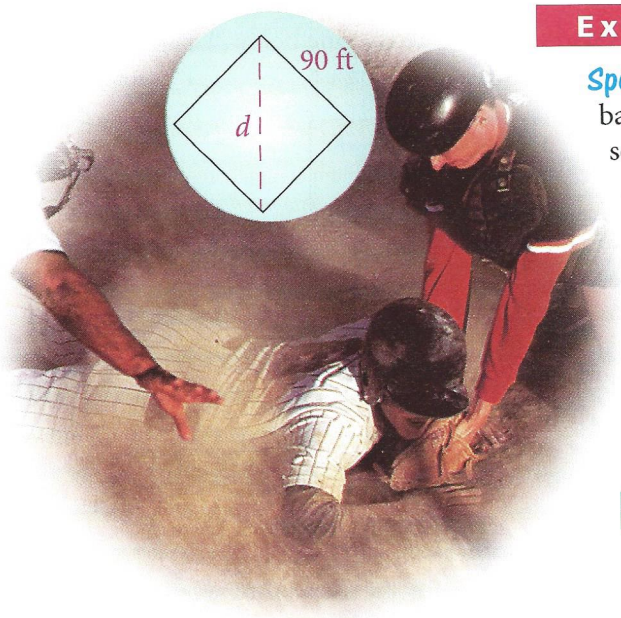
The pattern you observed in the Work Together (and generalized in Question 3) is the basis of the following theorem.

Theorem 5-6
45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, both legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

$$\text{hypotenuse} = \sqrt{2} \cdot \text{leg}$$





Example 1 **Relating to the Real World**

Sports A baseball diamond is a square. The distance from base to base is 90 ft. To the nearest foot, how far does the second baseman throw a ball to home plate?

The distance d from second base to home plate is the length of the hypotenuse of a 45°-45°-90° triangle.

$d = 90\sqrt{2}$ hypotenuse $= \sqrt{2} \cdot$ leg

90 × 2 √ = 127.27922

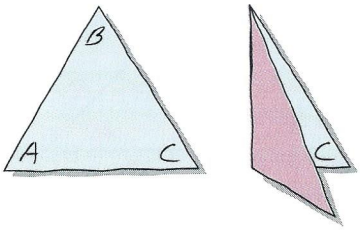
The distance from second base to home plate is about 127 ft.

5. a. Calculator Find $\sqrt{2}$ to the nearest thousandth.

b. Mental Math Use the answer to part (a) to estimate the length of a diagonal of a square with sides 100 ft long.

WORK TOGETHER

Work with a group.

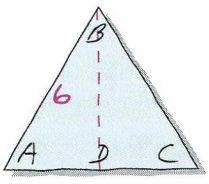


- Draw an equilateral triangle with sides 6 cm long and cut it out. Label the vertices A, B, and C. Fold vertex A onto vertex C as shown at the left. Unfold the triangle and label the fold-line $\overline{BD}$.
- With your group, make a list of everything you know about $\triangle ABC$, $\triangle ABD$, and $\triangle CBD$, their angles and their sides.

6. Name a pair of congruent triangles.

7. a. Find $m\angle A$, $m\angle ADB$, and $m\angle ABD$.

b. Name $\triangle ABD$ using its angle measures.



8. a. Complete: $\overline{DB}$ is the ___?___ of $\overline{AC}$.

b. If $AB = 6$, what is AD?

c. Use the Pythagorean Theorem to find BD in simplest radical form.

d. Find the ratios $\frac{AB}{AD}$ and $\frac{BD}{AD}$.

THINK AND DISCUSS

30°-60°-90° Triangles

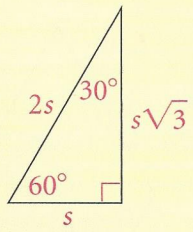
The ratios you found in Question 8 part (d) suggest the following theorem about 30°-60°-90° triangles.

Theorem 5-7
30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

hypotenuse = 2 · shorter leg

longer leg = $\sqrt{3}$ · shorter leg



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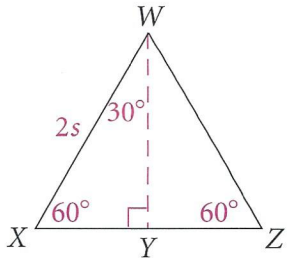


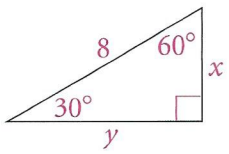
Refer to $\triangle WXZ$ at the left. Since $\overline{WY}$ is the perpendicular bisector of $\overline{XZ}$, $XY = \frac{1}{2}XZ$. That means that if $XW = 2s$, then $XY = s$.

$XY^2 + YW^2 = XW^2$	Use the Pythagorean Theorem.
$s^2 + YW^2 = (2s)^2$	Substitute s for XY and $2s$ for XW.
$YW^2 = 4s^2 - s^2$	Subtract s^2 from each side.
$YW^2 = 3s^2$	
$YW = s\sqrt{3}$	Find the square root of each side.

Example 2

Algebra Find the value of each variable.

a.



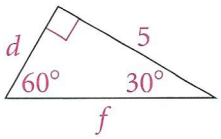
a. $8 = 2x$ — hypotenuse = 2 · shorter leg

$x = 4$

$y = x\sqrt{3}$ — longer leg = $\sqrt{3}$ · shorter leg

$y = 4\sqrt{3}$ — Substitute 4 for x.

b.



b. $5 = d\sqrt{3}$ — longer leg = $\sqrt{3}$ · shorter leg

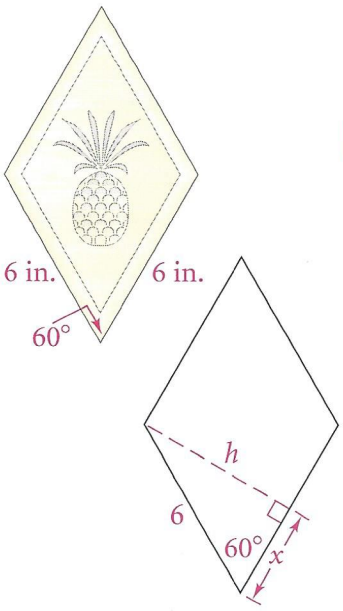
$d = \frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$ — Simplify.

$f = 2d$ — hypotenuse = 2 · shorter leg

$f = 2 \cdot \frac{5\sqrt{3}}{3} = \frac{10\sqrt{3}}{3}$ — Substitute $\frac{5\sqrt{3}}{3}$ for d.

9. Try This The shorter leg of a 30°-60°-90° triangle has length $\sqrt{6}$. What are the lengths of the other two sides? Leave your answers in simplest radical form.

You can use the properties of 30°-60°-90° triangles to find the dimensions you need to calculate area.



Example 3 **Relating to the Real World**

Design The rhombus at the left is a glass panel for a door. How many square inches of colored glass will you need for the panel?

Draw an altitude of the rhombus. Label x and h as shown.

$6 = 2x$ hypotenuse = 2 · shorter leg

$x = 3$

$h = 3\sqrt{3}$ longer leg = $\sqrt{3}$ · shorter leg

Use the value of h to find the area.

$A = bh$ Use the formula for area of a parallelogram.

$= 6(3\sqrt{3})$ Substitute 6 for b and $3\sqrt{3}$ for h.

6 × 3 × 3 √ = 31.176915

You will need about 31.2 in.^{2} of colored glass.

Exercises ON YOUR OWN

Find the value of each variable. Leave your answer in simplest radical form.

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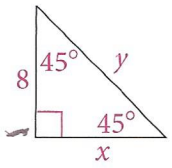

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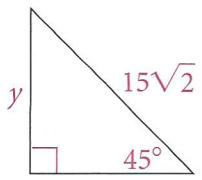

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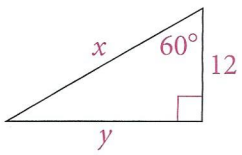

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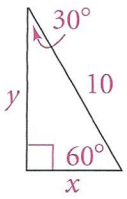

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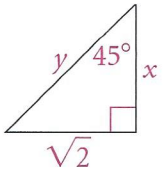

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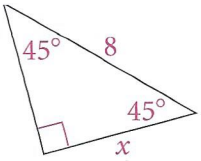

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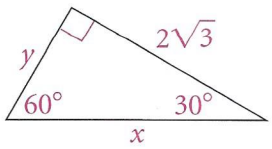

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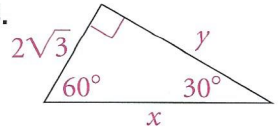
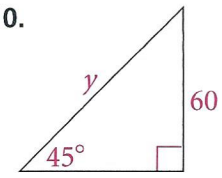


9. a. **Farming** A conveyor belt carries bales of hay from the ground to the loft of a barn 27.5 ft above ground. The belt makes a 30° angle with the ground. How far does a bale of hay travel on the conveyor belt?

b. The conveyor belt moves at 100 ft/min. How long does it take for a bale of hay to go from the ground to the barn loft?

Find the value of each variable. Leave your answer in simplest radical form.

10.



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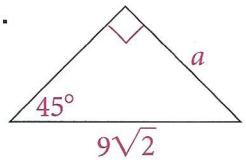

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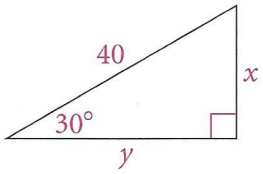

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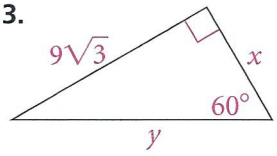

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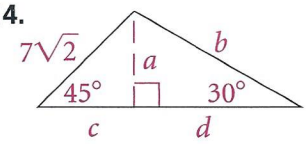

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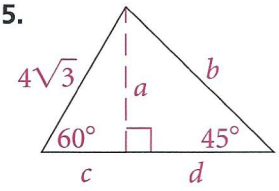

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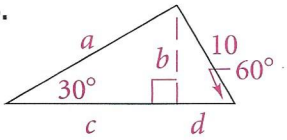

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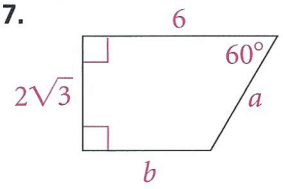
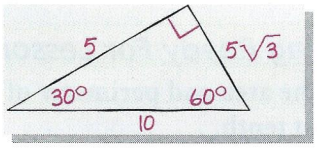


18. Writing Sandra drew this triangle. Rika said that the lengths couldn't be correct. With which student do you agree? Explain.



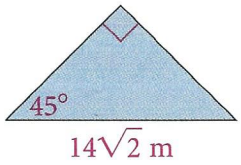
19. Standardized Test Prep Which of the following *cannot* be the lengths of sides of a 30°-60°-90° triangle?

A. $\frac{1}{2}, 1, \frac{\sqrt{3}}{2}$ **B.** $\sqrt{3}, 2\sqrt{3}, 3$ **C.** $1, \frac{1}{2}, \sqrt{3}$

D. $2\sqrt{2}, \sqrt{2}, \sqrt{6}$ **E.** $2, 4, 2\sqrt{3}$

Calculator Find the area of each figure. When an answer is not a whole number, round to the nearest tenth.

20.



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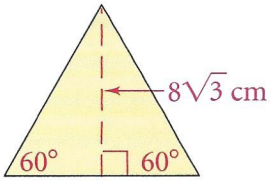

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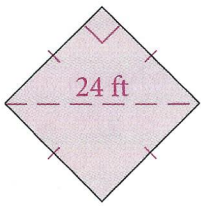

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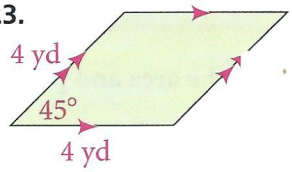

24. Helicopters The blades of a helicopter meet at right angles and are all the same length. The distance between the tips of two consecutive blades is 36 ft. How long is each blade? Round your answer to the nearest tenth.

25. Open-ended The hypotenuse of a 30°-60°-90° triangle is 12 ft long. Write a real-life problem that you can solve using this triangle. Show your solution.

26. A rhombus has a 60° angle and sides 5 cm long. What is its area? Round your answer to the nearest tenth.