5-2 Areas of Parallelograms and Triangles

What You'll Learn

- Finding areas of parallelograms and triangles

...And Why

To solve design problems in architecture and landscaping

What You'll Need

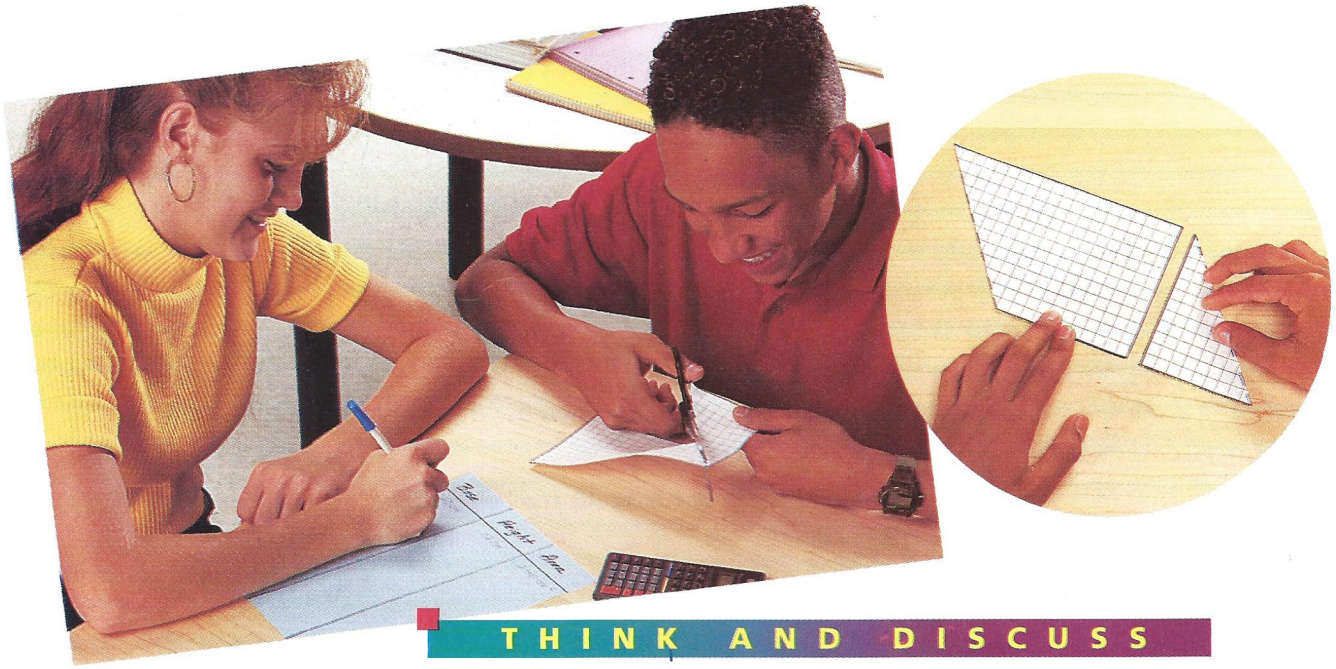
- centimeter grid paper
- straightedge
- scissors
- tape

WORK TOGETHER

Have each member of your group cut out a different rectangle from centimeter grid paper.

- Record the base, height, and area of each rectangle.
- Cut out a triangle from one side of the rectangle as shown below. Tape it to the opposite side to form a parallelogram.

1. Compare each original rectangle with the parallelogram formed. With your group, list all the ways the rectangle and the parallelogram are the same and all the ways they are different.



THINK AND DISCUSS

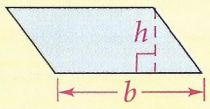
Areas of Parallelograms

In the Work Together, you cut a rectangle into two pieces and used the pieces to form another parallelogram. The area of the parallelogram was the same as the area of the rectangle. This suggests the following theorem.

Theorem 5-2
Area of a Parallelogram

The area of a parallelogram is the product of any base and the corresponding height.

$A = bh$



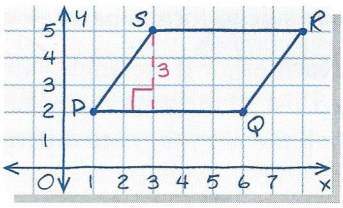
You can choose any side to be a **base** of a parallelogram. An **altitude** is any segment perpendicular to the line containing the base drawn from the side opposite the base. The **height** is the length of the altitude.

2. Draw any parallelogram and draw altitudes to two adjacent sides.

Example 1

Coordinate Geometry What is the area of $\square PQRS$ with vertices $P(1, 2)$, $Q(6, 2)$, $R(8, 5)$, and $S(3, 5)$?

Graph $\square PQRS$. If you choose $\overline{PQ}$ as the base, then the height is 3.

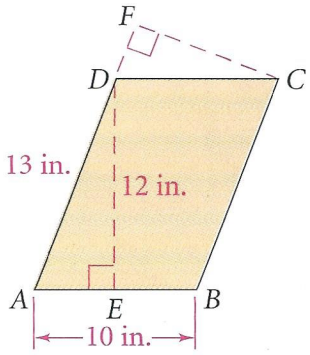


$b = PQ = 5$
$h = 3$
$A = bh = 5(3)$
$= 15$

$\square PQRS$ has area 15 square units.

3. Try This What is the area of $\square EFGH$ with vertices $E(-4, 3)$, $F(0, 3)$, $G(1, -2)$, and $H(-3, -2)$?

You can use the area formula to find missing dimensions in a parallelogram.



Example 2

In $\square ABCD$, $\overline{DE}$ and $\overline{CF}$ are altitudes. Find CF to the nearest tenth.

Find the area of $\square ABCD$. Then use the area formula to find CF.

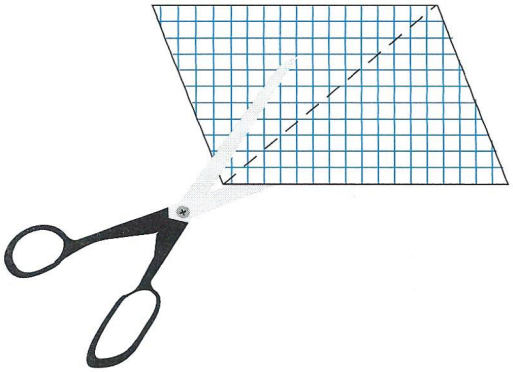
$A = bh$
$= 10(12)$ **Use base *AB* and height *DE*.**
$= 120$

The area of $\square ABCD$ is 120 in.^{2}.

$A = bh$
$120 = 13(CF)$ **Use base *AD* and height *CF*.**
$CF = \frac{120}{13}$ **Divide each side by 13.**
≈ 9.2

$\overline{CF}$ is about 9.2 in. long.

4. Try This A parallelogram has sides 15 cm and 18 cm. The altitude perpendicular to the line containing the 15 cm side is 9 cm long. Sketch the parallelogram. Then find the length of the altitude perpendicular to the line containing the 18-cm side.



WORK TOGETHER

Work in groups. Have each member of your group cut out a different parallelogram from centimeter grid paper.

- Record the base, height, and area of each parallelogram.
- Cut each parallelogram along a diagonal as shown, forming two triangles.

5. How does the area of each triangle compare to the area of the parallelogram?

THINK AND DISCUSS

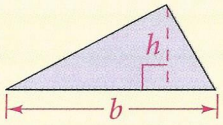
Areas of Triangles

In the Work Together, you cut a parallelogram into two congruent triangles of equal area. This suggests the following theorem.

Theorem 5-3
Area of a Triangle

The area of a triangle is half the product of any base and the corresponding height.

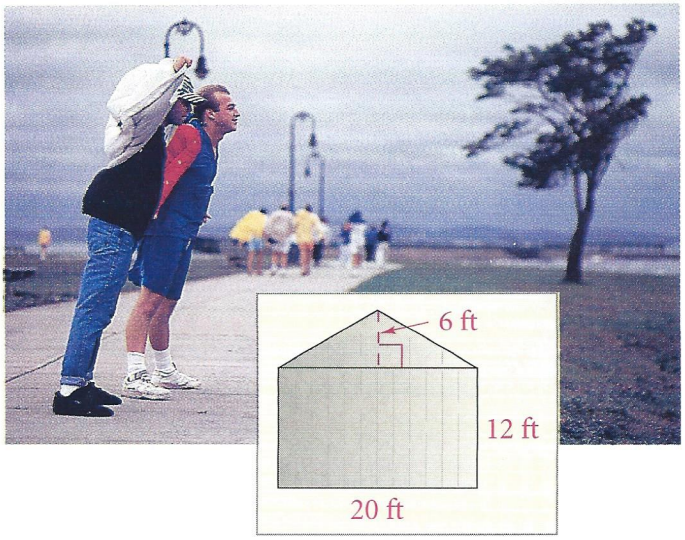
$$A = \frac{1}{2}bh$$



You can choose any side to be a **base** of a triangle. The corresponding **height** is the length of an altitude drawn to the line containing that base.

Example 3 **Relating to the Real World**

Architecture When designing a building, an architect must be sure that the building can stand up to hurricane force winds, which have a velocity of 73 mi/h or more. The formula $F = 0.004Av^2$ gives the force F in pounds exerted by a wind blowing against a flat surface. A is the area of the surface in square feet, and v is the wind velocity in miles per hour. How much force is exerted by a 73 mi/h wind blowing directly against the side of this building?



Find the area of the side of the building.

$$\text{triangle area} = \frac{1}{2}bh = \frac{1}{2}(20)6 = 60\text{ ft}^2$$

$$\text{rectangle area} = bh = 20(12) = 240\text{ ft}^2$$

$$\text{area of end of building} = 60 + 240 = 300\text{ ft}^2$$

$F = 0.004Av^2$ **Use the formula for force.**

$= 0.004(300)(73)^2$ **Substitute 300 for *A* and 73 for *v*.**

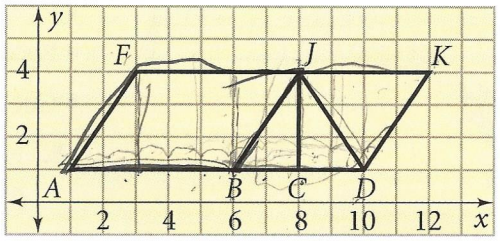
$= 6394.8$

The force is about 6400 lb.

Exercises ON YOUR OWN

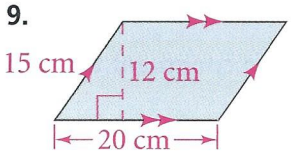
Find the area of each figure.

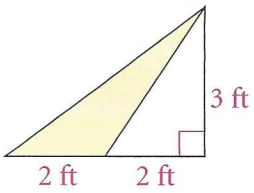
1. $\square ABJF$
2. $\triangle BDJ$
3. $\triangle DKJ$
4. $\square BDKJ$
5. $\square ADKF$
6. $\triangle BCJ$

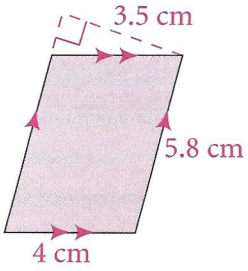


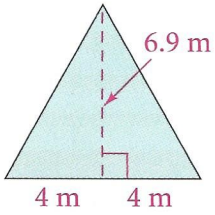
7. The area of a parallelogram is 24 in.^{2} and the height is 6 in. Find the length of the base.
8. An isosceles right triangle has area of 98 cm^2. Find the length of each leg.

Find the area of each shaded region.

9. 

10. 

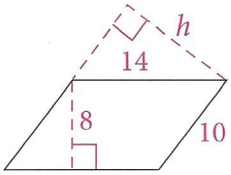
11. 

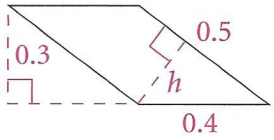
12. 

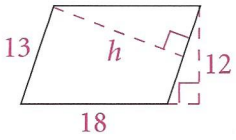
Coordinate Geometry **(a) Graph the lines. (b) Find the area of the triangle enclosed by the lines.**

13. $y = x$, $x = 0$, and $y = 7$
14. $y = x + 2$, $y = 2$, $x = 6$
15. $y = -\frac{1}{2}x + 3$, $y = 0$, $x = -2$

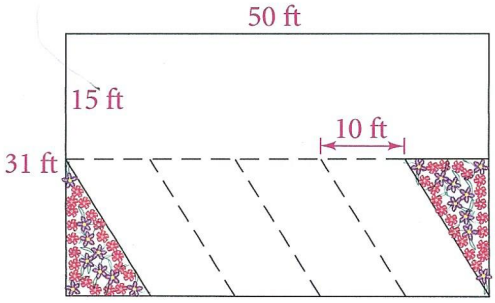
Find the value of h in each parallelogram.

16. 

17. 

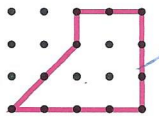
18. 

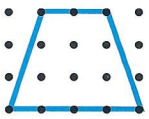
19. **Landscaping** Taisha's Bakery has a plan for a 50 ft-by-31 ft parking lot. The four parking spaces are congruent parallelograms, the driving area is a rectangle, and the two unpaved areas for flowers are congruent triangles.
    a. **Writing** Explain two different ways to find the area of the region that must be paved.
    b. **Verify** your answer to part (a) by using each method to find the area.

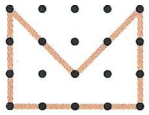


20. **Algebra** In a triangle, a base and the corresponding height are in the ratio 3 : 2. The area is 108 in.^{2}. Find the base and the corresponding height.

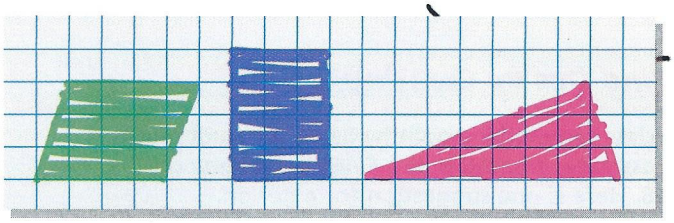
Find the area of each figure.

21. 

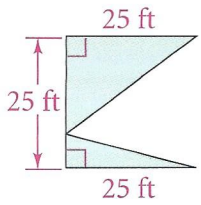
22. 

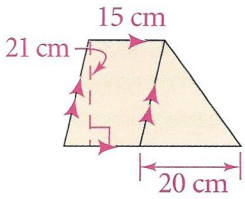
23. 

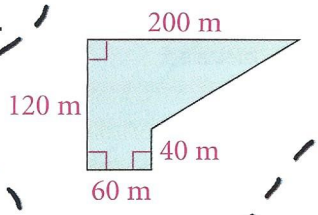
24. **Probability** Ann drew these three figures on a grid. A fly landed at random at a point on the grid.
   a. Is the fly more likely to have landed on one of the figures or on the blank grid? Explain.
   b. Suppose you know the fly landed on one of the figures. Is the fly more likely to have landed on one figure than on another? Explain.



Find the area of each figure.

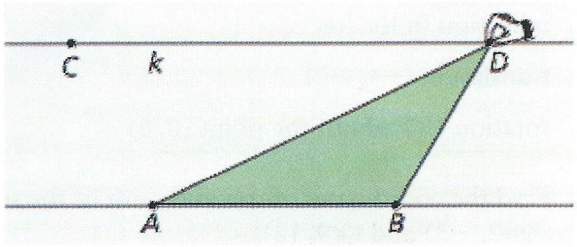
25. 

26. 

27. 

28. **Open-ended** Using graph paper, draw an acute triangle, an obtuse triangle, and a right triangle, each with area 12 units^2.

29. **Technology** Juanita used geometry software to create the figure at the right. She drew segment $\overline{AB}$, chose point C, and constructed line k parallel to $\overline{AB}$ through point C. Then Juanita chose point D on line k. Next she dragged point D along line k to form different triangles. How do the areas of the triangles compare? Explain.



The ancient Greek mathematician Heron is most famous for his formula for the area of a triangle in terms of its sides *a*, *b*, and *c*.

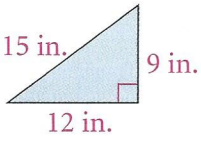
$$A = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{1}{2}(a+b+c)$$

Use Heron's formula and a calculator to find the area of each triangle. Round your answer to the nearest whole number.

30. $a = 8$ in., $b = 9$ in., $c = 10$ in.

31. $a = 15$ m, $b = 17$ m, $c = 21$ m

32. a. Use Heron's formula to find the area of the triangle at the right.
   b. **Verify** your answer to part (a) by using the formula $A = \frac{1}{2}bh$.



Coordinate Geometry **The vertices of a polygon are given. Graph each polygon and find its area.**

33. $A(3, 9)$, $B(8, 9)$, $C(2, -3)$, $D(-3, -3)$

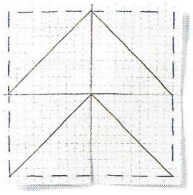
34. $E(1, 1)$, $F(4, 5)$, $G(11, 5)$, $H(8, 1)$

35. $M(-2, -5)$, $L(1, -5)$, $N(2, -2)$

36. $R(1, 2)$, $S(1, 6)$, $T(4, 1)$

Chapter Project **Find Out by Creating**

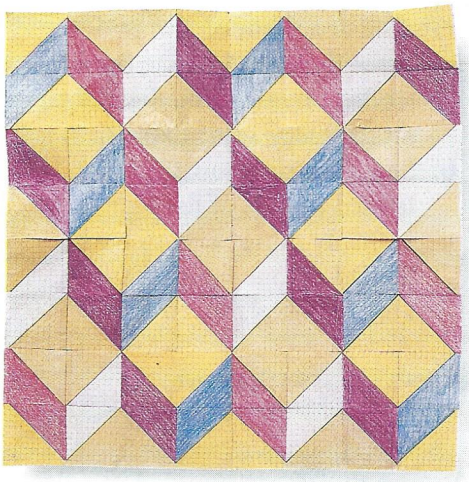
Your class can model a quilt by using the quilt blocks your classmates created in the *Find Out* activity on page 248. Here is one suggestion for a design.



On each block, mark off a $\frac{1}{4}$-in. border for seams. Draw the four diagonals pictured.

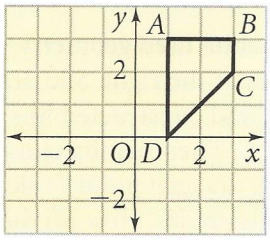
Staple four blocks together in a row, keeping the orientation shown at the left throughout the row. Do this until you have four rows.

Staple the rows together, turning the second and fourth row upside down. Color the blocks to create a three-dimensional illusion.



Exercises MIXED REVIEW

Transformations **Find the coordinates of the images of *A*, *B*, *C*, and *D* after each transformation.**



37. reflection in the line $x = 1$

38. translation $\langle -4, -7 \rangle$

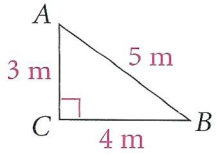
39. rotation 180° about the point $(0, 0)$

40. Find the coordinates of the midpoint of the segment joining $P(-2, -3)$ and $Q(9, 12)$.

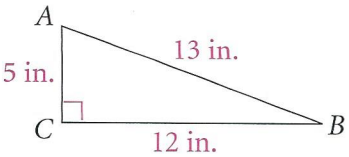
Getting Ready for Lesson 5-3

Square the lengths of the sides of each triangle. What do you notice?

41.



42.



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