Connections Animal Science . . . and more

5-1 Understanding Perimeter and Area

What You'll Learn

- Finding area and perimeter of squares and rectangles

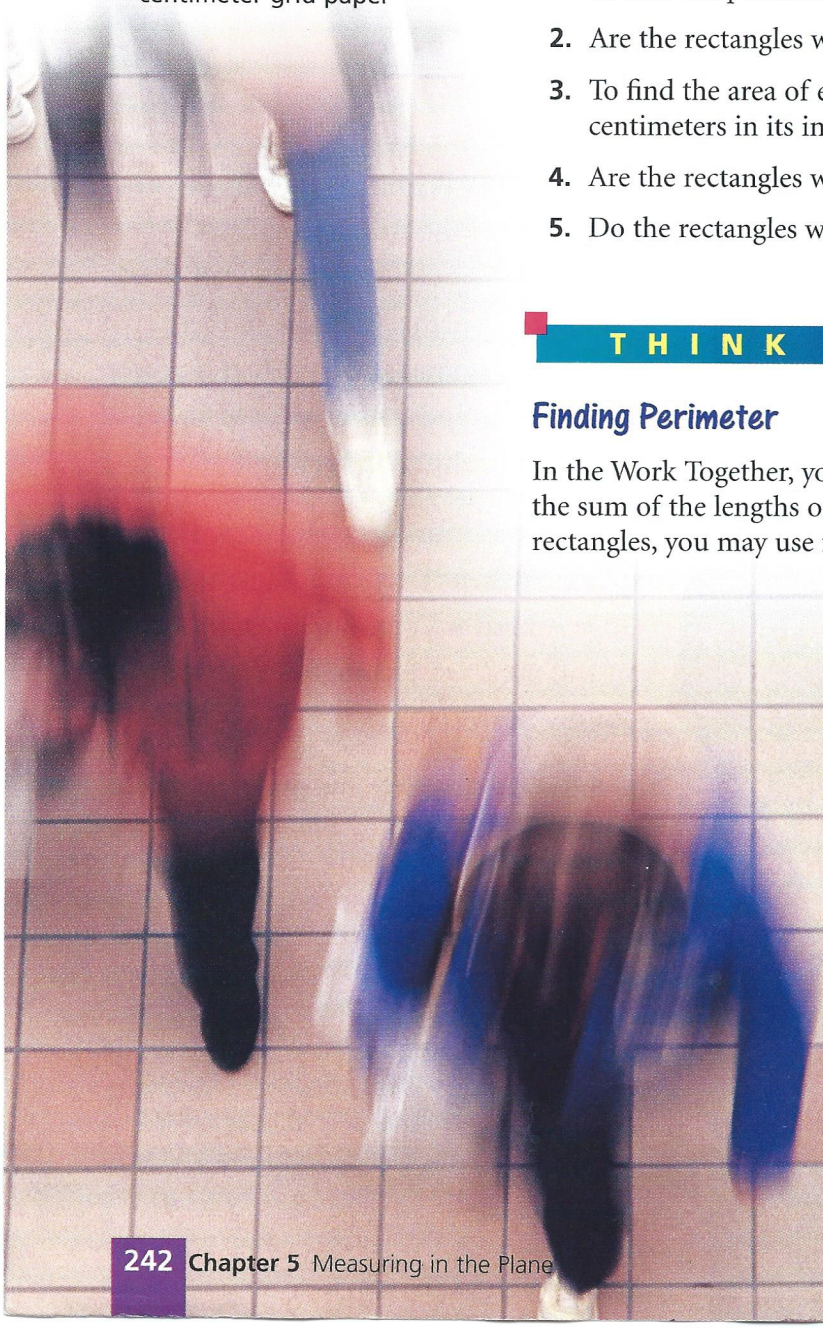
...And Why

To find the perimeters of banners, animal pens, and gardens

To find the surface area to be covered by carpet or by tiles

What You'll Need

- centimeter grid paper



WORK TOGETHER

In your group, draw each figure on centimeter grid paper.

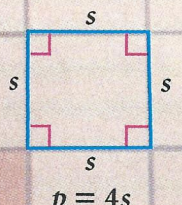
- a rectangle with length 5 cm and width 3 cm
- a rectangle with base 8 cm and height 2 cm
- a rectangle with each side 4 cm

1. Record the perimeter of each rectangle.
2. Are the rectangles with equal perimeters congruent, similar, or neither?
3. To find the area of each rectangle, count the number of square centimeters in its interior. Record the area of each rectangle.
4. Are the rectangles with the same area congruent, similar, or neither?
5. Do the rectangles with equal perimeters have the same area?

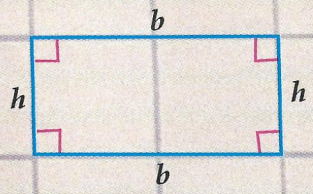
THINK AND DISCUSS

Finding Perimeter

In the Work Together, you found the **perimeter of a polygon** by finding the sum of the lengths of the sides. In special cases such as squares and rectangles, you may use formulas for perimeter.



$p = 4s$

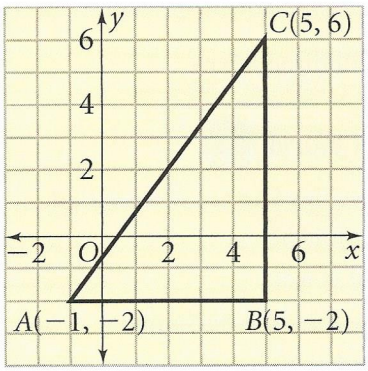


$p = 2b + 2h$ or $p = 2(b + h)$

6. Can you use the formula for the perimeter of a square to find the perimeter of any rectangle? Explain.

You can use the Distance Formula to find perimeter in the coordinate plane.

Example 1



Coordinate Geometry Find the perimeter of $\triangle ABC$.

Find the length of each side. Add the lengths to find the perimeter.

$AB = 5 - (-1) = 6$

$BC = 6 - (-2) = 8$

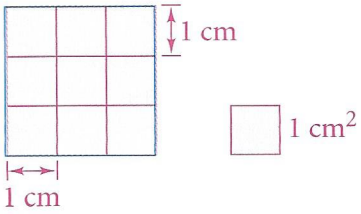
$AC = \sqrt{(5 - (-1))^2 + (6 - (-2))^2}$ **Use the Distance Formula.**

$= \sqrt{6^2 + 8^2} = \sqrt{100} = 10$

$AB + BC + AC = 6 + 8 + 10 = 24$

The perimeter of $\triangle ABC$ is 24 units.

7. **Try This** Graph the quadrilateral with vertices $K(-3, -3)$, $L(1, -3)$, $M(1, 4)$, and $N(-3, 1)$. Find the perimeter of $KLMN$.

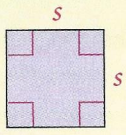


Finding Area

The **area of a polygon** is the number of square units enclosed by the polygon. The blue square at the left encloses nine smaller red squares. Each red square has sides 1 cm long and is called a square centimeter. By counting square centimeters, you see that the blue square has area 9 cm^2.

Postulate 5-1 The area of a square is the square of the length of a side.

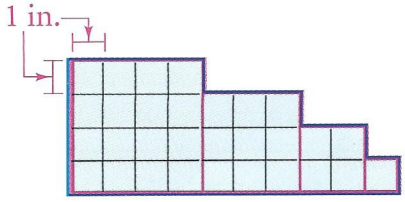
$A = s^2$



Postulate 5-2 If two figures are congruent, their areas are equal.

Postulate 5-3 The area of a region is the sum of the areas of its nonoverlapping parts.

8. a. **Try This** What is the area of a square whose sides are 12 in. long?
   b. What is the area of a square whose sides are 1 ft long?
   c. How many square inches are in a square foot?



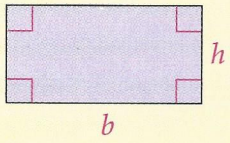
9. a. By counting squares, find the area of the polygon outlined in blue.
   b. Use Postulate 5-1 to find the area of each square outlined in red.
   c. How does the sum of your answers to part (b) compare to your answer to part (a)? Which postulate does this **verify?**

You can select any side of a rectangle to be the base. Because adjacent sides are perpendicular, the length of a side adjacent to the base is the height.

Theorem 5-1
Area of a Rectangle

The area of a rectangle is the product of its base and height.

$A = bh$



To find area, you must use the same units for all dimensions.

Example 2 **Relating to the Real World**

Design You are designing a rectangular banner. The banner will be 2 yd long and 4 ft wide. How much material will you need?

$2 \text{ yd} = 6 \text{ ft}$ — Change units using 1 yd = 3 ft.

$A = bh$ — Use the formula for the area of a rectangle.

$= 6(4) = 24$ — Substitute 6 for b and 4 for h.

The area of the banner is 24 ft^2. You will need at least 24 ft^2 of material.

PROBLEM SOLVING

Look Back Find the area of the banner in Example 2 by first changing all units to yards.

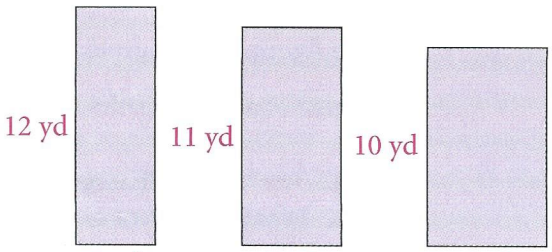
10. Try This Find the area of a rectangle with length 75 cm and width 2 m.

You can use a graphing calculator or spreadsheet technology to find maximum and minimum values for area and perimeter problems.

Example 3 **Relating to the Real World**

Animal Science You have 32 yd of fencing. You want to make a rectangular pen for a calf you are raising for a 4-H project. What are the dimensions of the rectangle that will result in the maximum area? What is the maximum area?

Draw some possible rectangular pens and find their areas.

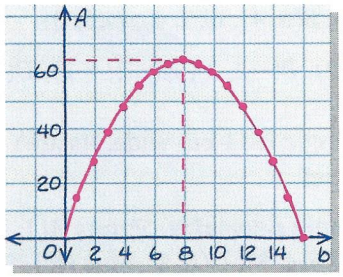


$A = 48 \text{ yd}^2$ $A = 55 \text{ yd}^2$ $A = 60 \text{ yd}^2$

	A	B	C
1	b	h = 16 − b	A = bh
2	1	15	15
3	2	14	28
4	3	13	39
5	...	...	...

Create a spreadsheet to find area. Choose values for b from 0 to 15.

Make a graph of the spreadsheet values. Graph values of b on the horizontal axis and values of A on the vertical axis. Connect the points with a smooth curve.



The maximum value occurs at $b = 8$. When $b = 8$, $h = 8$ and $A = 64$.

To have the maximum area for your calf, you should fence a square with sides 8 yd long. The maximum area is 64 yd^2.

11. **Critical Thinking** Show how the equation $h = 16 - b$ was derived from the formula $2b + 2h = 32$.

12. Use the answer to Example 3. Make a **conjecture** by completing this statement: If you have a fixed amount of fencing to enclose a rectangle, you can get the maximum area by enclosing a ___?___.

Exercises ON YOUR OWN

Estimation **Estimate the perimeter of each item.**

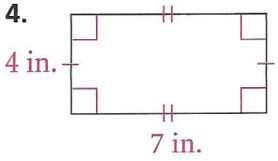
1. the cover of this book

2. the cover of your notebook

3. a classroom bulletin board

Mental Math **Find the perimeter of each figure.**

4.



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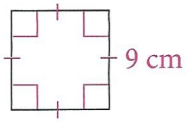

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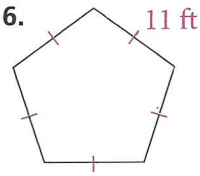

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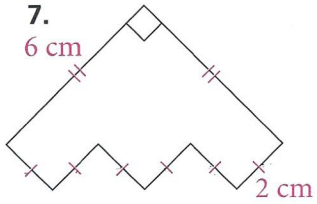


Find the perimeter of each rectangle with the given base and height.

8. 21 in., 7 in.

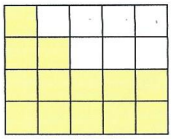
9. 16 cm, 23 cm

10. 24 m, 36 m

11. 14 ft, 23 ft

The figures below are drawn on centimeter graph paper. Find the area of the shaded portion of each figure.

12.



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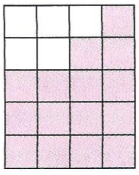

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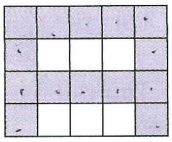

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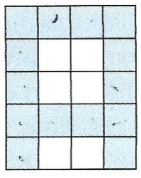

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Coordinate Geometry **Graph each rectangle *ABCD* and find its area.**

16. $A(0, 0)$, $B(0, 4)$, $C(5, 4)$, $D(5, 0)$

17. $A(1, 4)$, $B(1, 7)$, $C(5, 7)$, $D(5, 4)$

18. $A(-3, 2)$, $B(-2, 2)$, $C(-2, -2)$, $D(-3, -2)$

19. $A(-2, -6)$, $B(-2, -3)$, $C(3, -3)$, $D(3, -6)$

20. A rectangle is 11 cm wide. Its area is 176 cm^2. What is the length of the rectangle?

21. **Coordinate Geometry** Points $A(1, 1)$, $B(10, 1)$, $C(10, 8)$, $D(7, 8)$, $E(7, 5)$, $F(4, 5)$, $G(4, 8)$, and $H(1, 8)$ are the coordinates of the vertices of polygon *ABCDEFGH*.

a. Draw the polygon on graph paper.

b. Find the perimeter of the polygon.

c. Divide the polygon into rectangles.

d. Find the area of the polygon.

22. The perimeter of a rectangle is 40 cm and the base is 12 cm. What is the area?

23. A square and a rectangle have equal areas. The rectangle is 64 cm by 81 cm. What is the perimeter of the square?

Find the area of each rectangle with the given base and height.

24. 4 ft 6 in., 4 in.

25. 1 yd 18 in., 4 yd

26. 2 ft 3 in., 6 in.

Building Safe Stairs

Since falls are a major cause of injury, it makes sense to be concerned about the safety of stairs. According to John Templer, the world's foremost authority on stairs, steps with a 7-in. riser and an 11-in. tread form the safest possible stairs.

Prior to his investigations, Francois Blondel's formula, dated 1675, had recommended that stair measurements conform to the formula 2(riser) + tread = 25.5 in.

Source: *Smithsonian*

27. **Carpeting** You use John Templer's dimensions to build a stairway with six steps. You want to carpet the stairs with a 3-ft wide runner from the bottom of the first riser to the top of the sixth riser.

a. Find the area of the runner.

b. Since a roll of carpet is 12 ft across, a rectangle of carpet that measures 3 ft by 12 ft is cut from the roll to make the runner. How many square feet of the material will be wasted?

c. The carpet costs \$17.95/yd^2. You must pay for the entire piece that is cut. Find the cost of the carpet.

d. Binding for the edge of the runner costs \$1.75/yd. How much will the binding cost if the two long edges of the runner are bound?

Writing **Tell whether you need to know area or perimeter in order to determine how much of each item to buy. Explain your choice.**

28. edging for a garden

29. paint for a basement floor

30. wallpaper for a bedroom

31. weatherstripping for a door

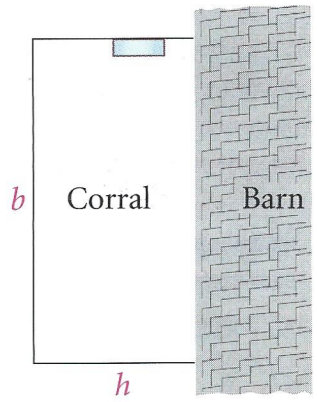
32. Tiling The Art Club is tiling an 8 ft-by-16 ft wall at the entrance to the school. They are creating a design by using different colors of 4 in.-by-4 in. tiles. How many tiles do the students need?

33. Gardening You want to make a 900-ft^2 rectangular garden to grow corn. In order to keep raccoons out of your corn, you must fence the garden. You want to use the minimum amount of fencing so that your costs will be as low as possible.

a. List some possible dimensions for the rectangular garden. Find the perimeter of each rectangle.

b. Technology Create a spreadsheet listing integer values of b and the corresponding values of h and P. What dimensions will give you a garden with the minimum perimeter?

34. You want to build a rectangular corral by using one side of a barn and fencing the other three sides. You have enough material to build 100 ft of fence.



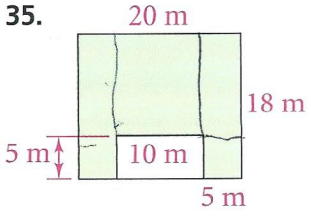
a. Technology Create a spreadsheet listing integer values of b and the corresponding values of h and A.

b. Coordinate Geometry Make a graph using your spreadsheet values. Graph b on the horizontal axis and A on the vertical axis.

c. Describe the dimensions of the corral with the greatest area.

Find the area of the shaded portion of each figure. All angles in the figures are right angles.

35.



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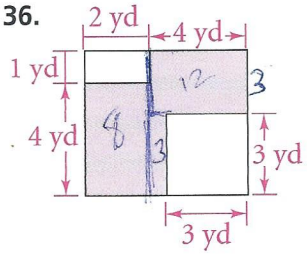

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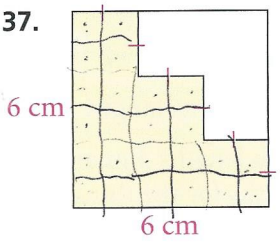

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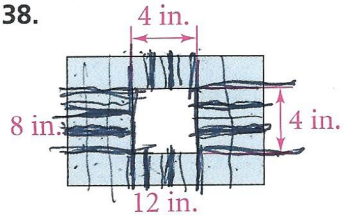

Coordinate Geometry **Graph each quadrilateral $ABCD$. Find its perimeter.**

39. $A(-2, 2)$, $B(0, 2)$, $C(4, -1)$, $D(-2, -1)$

40. $A(-4, -1)$, $B(4, 5)$, $C(4, -2)$, $D(-4, -2)$

41. $A(0, 1)$, $B(3, 5)$, $C(5, 5)$, $D(5, 1)$

42. $A(-5, 3)$, $B(7, -2)$, $C(7, -6)$, $D(-5, -6)$

43. Open-ended The area of a 5 in.-by-5 in. square is the same as the sum of the areas of a 3 in.-by-3 in. square and a 4 in.-by-4 in. square. Find two or more squares that have the same total area as an 11 in.-by-11 in. square.

PROBLEM SOLVING HINT

Make a list of perfect squares.