1-4 Measuring Angles and Segments

What You'll Learn

- Finding the length of a segment and the measure of an angle

...And Why

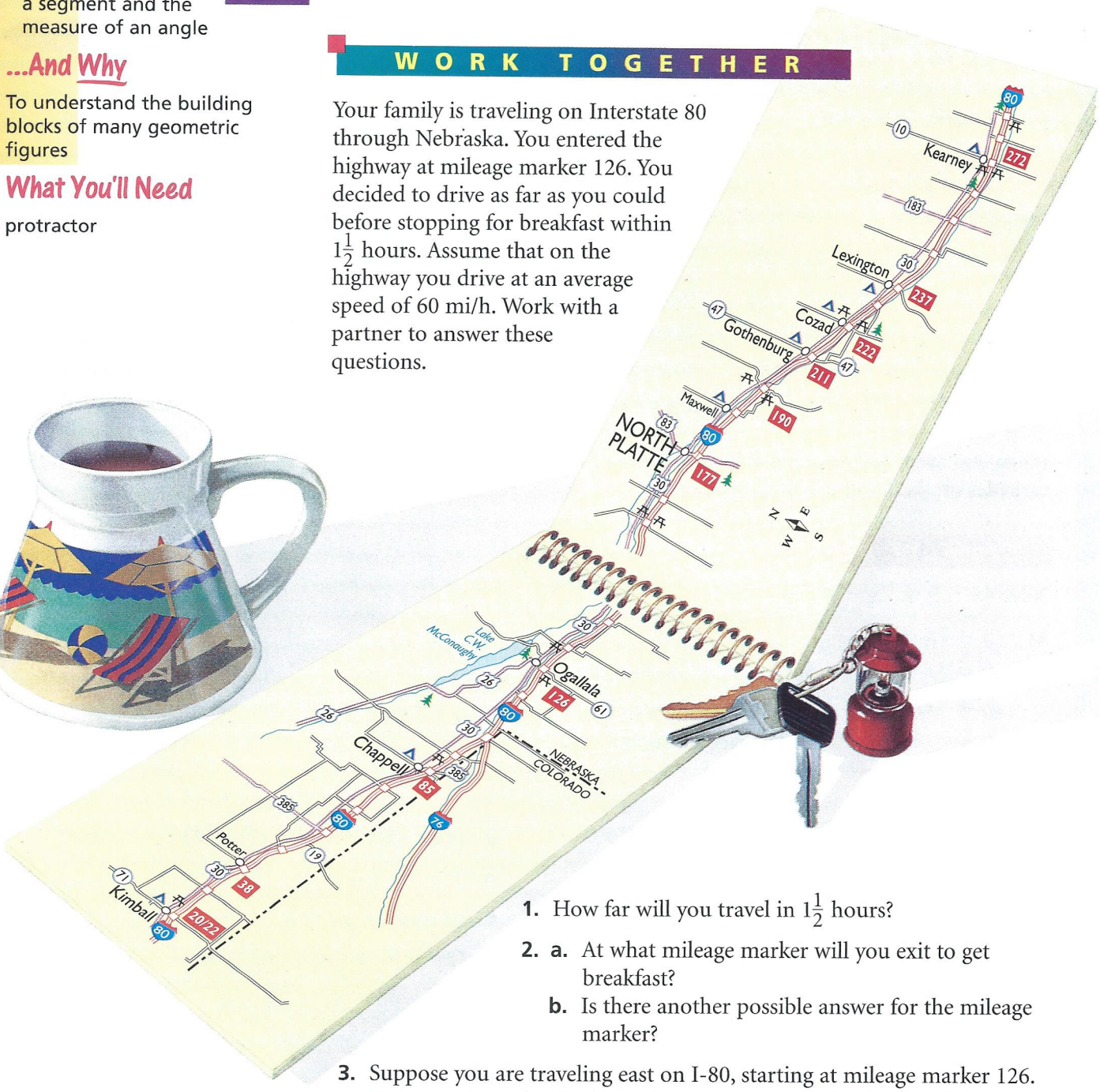
To understand the building blocks of many geometric figures

What You'll Need

protractor

WORK TOGETHER

Your family is traveling on Interstate 80 through Nebraska. You entered the highway at mileage marker 126. You decided to drive as far as you could before stopping for breakfast within $1\frac{1}{2}$ hours. Assume that on the highway you drive at an average speed of 60 mi/h. Work with a partner to answer these questions.



1. How far will you travel in $1\frac{1}{2}$ hours?

2. **a.** At what mileage marker will you exit to get breakfast?
   b. Is there another possible answer for the mileage marker?

3. Suppose you are traveling east on I-80, starting at mileage marker 126. Not counting side trips for sightseeing, you travel 111 mi on I-80 before you exit for a campground.
   a. Do mileage markers increase or decrease from west to east?
   b. What mileage marker is at your exit?

4. Does the *direction* you travel affect the *distance* you travel?

THINK AND DISCUSS

Measuring Segments

If you picture straightening out the map of Interstate 80, you will have a model for a number line. The mileage markers represent *coordinates*.

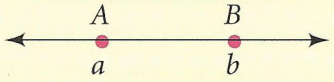
Postulate 1-5
Ruler Postulate

The points of a line can be put into a one-to-one correspondence with the real numbers so that the distance between any two points is the absolute value of the difference of the corresponding numbers.

the length of $\overline{AB}$

$$AB = |a - b|$$

coordinate of A coordinate of B



5. **Critical Thinking** Why do you think that absolute value is used to express the distance between two points?

Example 1

PROBLEM SOLVING HINT
Draw a diagram.

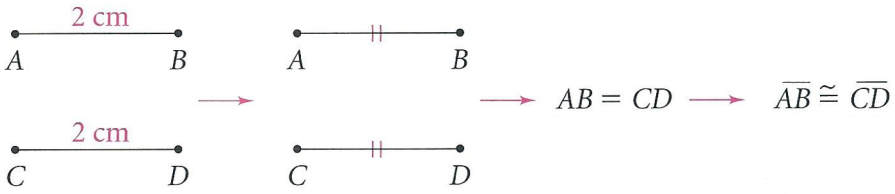
Find QS if the coordinate of Q is -3 and the coordinate of S is 21.

$QS = |-3 - 21| = |-24| = 24$

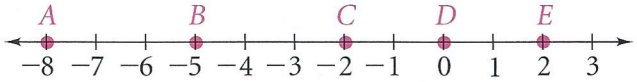
6. Suppose you subtracted -3 from 21 in Example 1. Would you get the same result? Why or why not?

7. **Try This** Find AB if the coordinate of point A is -8, and the coordinate of point B is 11.

Two segments with the same length are **congruent** ($\cong$). In other words, if $AB = CD$, then $\overline{AB} \cong \overline{CD}$. You can use these statements interchangeably. Segments can be marked alike to show that they are congruent.

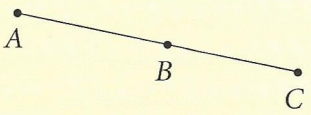


8. a. **Try This** Name two segments that are congruent.
   b. Name a second pair of congruent segments.



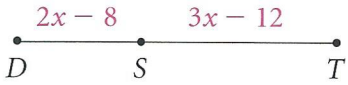
Postulate 1-6
Segment Addition Postulate

If three points A, B, and C are collinear and B is between A and C, then $AB + BC = AC$.



Example 2

Algebra If $DT = 60$, find the value of x. Then find DS and ST.



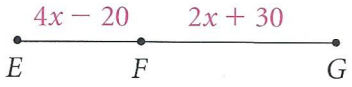
For more practice with solving linear equations, see Skills Handbook, page 663.

$DS + ST = DT$	**Segment Addition Postulate**
$(2x - 8) + (3x - 12) = 60$	**Substitution**
$5x - 20 = 60$	**Simplify.**
$5x = 80$	**Add 20 to each side.**
$x = 16$	**Divide each side by 5.**

$DS = 2x - 8 = 2(16) - 8 = 24$

$ST = 3x - 12 = 3(16) - 12 = 36$

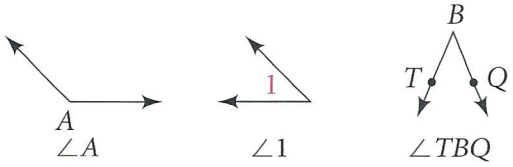
9. Explain how to check the answers in Example 2.

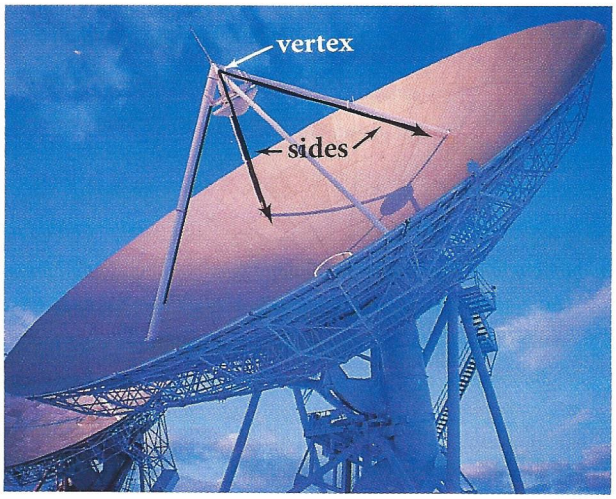


10. a. Try This $EG = 100$. Find the value of x.
b. Find EF and FG.

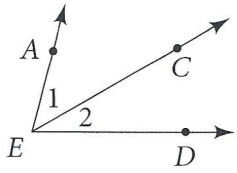
Measuring Angles

An **angle** ($\angle$) is formed by two rays (called *sides* of the angle) with the same endpoint (called the *vertex* of the angle). You can name an angle several ways.



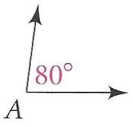


11. a. Name $\angle 1$ two other ways.
b. Name $\angle CED$ two other ways.
c. Would it be correct to refer to any of the angles at the right as $\angle E$? Why or why not?



Angles are measured in *degrees*. The *measure* of $\angle A$ is written as $m\angle A$.

$m\angle A = 80$



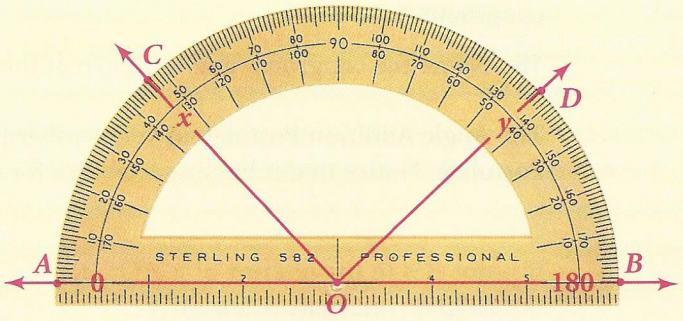
When you use a *protractor* to measure angles you are applying the following postulate.

Postulate 1-7
Protractor Postulate

Let $\overrightarrow{OA}$ and $\overrightarrow{OB}$ be opposite rays in a plane. $\overrightarrow{OA}$, $\overrightarrow{OB}$ and all the rays with endpoint O that can be drawn on one side of $\overleftrightarrow{AB}$ can be paired with the real numbers from 0 to 180 in such a way that:

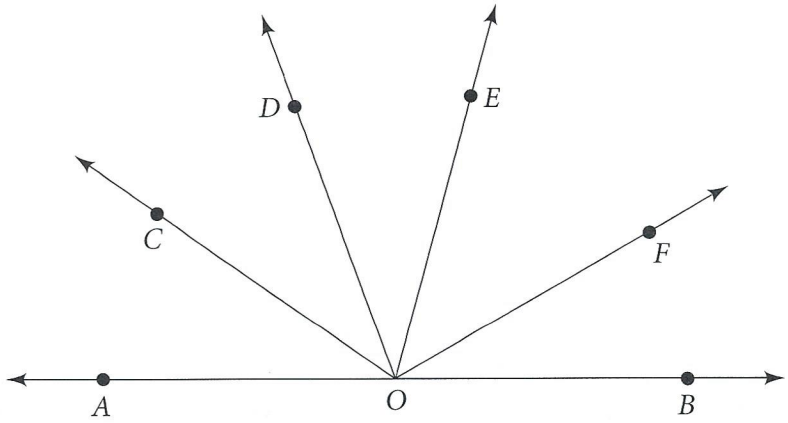
a. $\overrightarrow{OA}$ is paired with 0 and $\overrightarrow{OB}$ is paired with 180.

b. If $\overrightarrow{OC}$ is paired with x and $\overrightarrow{OD}$ is paired with y, then $m\angle COD = |x - y|$.



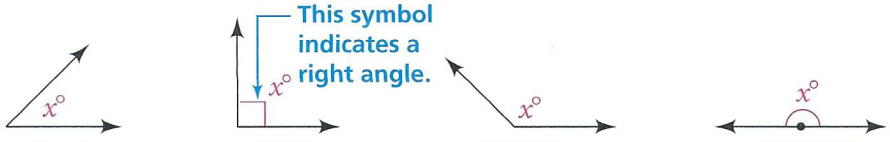
12. Try This Use a protractor to find the measure of each angle.

a. $m\angle AOC$ **b.** $m\angle EOB$ **c.** $m\angle DOF$ **d.** $m\angle COE$



13. Try This Use a protractor to draw a 105° angle.

You can classify angles according to their measures.



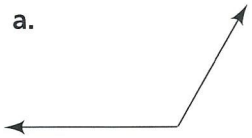
Acute
$0 < x < 90$

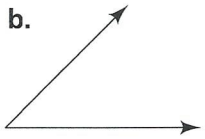
Right
$x = 90$

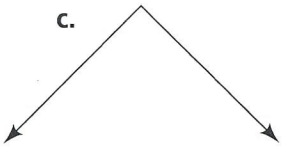
Obtuse
$90 < x < 180$

Straight
$x = 180$

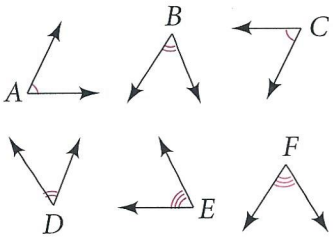
14. Estimate the measure of each angle. Then use a protractor to find the measure.

a. 

b. 

c. 

15. Classify each angle in Exercise 14 as *acute, obtuse,* or *right.*



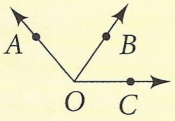
Angles with the same measure are **congruent.** In other words, if $m\angle 1 = m\angle 2$, then $\angle 1 \cong \angle 2$. You can use these statements interchangeably. Angles can be marked alike to show that they are congruent.

16. Name the congruent angles shown at the left.

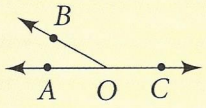
The Angle Addition Postulate is very similar to the Segment Addition Postulate. Notice that it has a special case for straight angles.

Postulate 1-8
Angle Addition Postulate

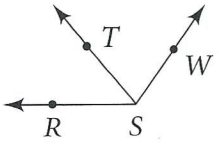
If point B is in the interior of $\angle AOC$, then $m\angle AOB + m\angle BOC = m\angle AOC$.



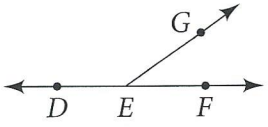
If $\angle AOC$ is a straight angle, then $m\angle AOB + m\angle BOC = 180$.



17. a. $m\angle RST = 50$ and $m\angle RSW = 125$. What is $m\angle TSW$?



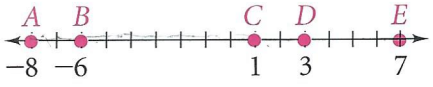
b. $m\angle DEG = 145$. What is $m\angle GEF$?



Exercises ON YOUR OWN

Complete each equation.

1. $AC =$ ■ **2.** $BD =$ ■ **3.** $AD =$ ■ **4.** $BE =$ ■



Exs. 1–10

Write *true* or *false.*

5. $\overline{AB} \cong \overline{ED}$ **6.** $BD < CD$ **7.** $AC + BD = AD$ **8.** $AC + CD = AD$

9. Name two pairs of congruent segments.

10. $EG = 5$. Find the coordinate of point G. Is there another possibility?

Use the figure at the right for Exercises 11–14.

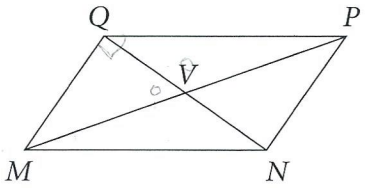
11. If $RS = 15$ and $ST = 9$, then $RT =$ ■.

12. If $ST = 15$ and $RT = 40$, then $RS =$ ■.

13. **Algebra** If $RS = 3x + 1$, $ST = 2x - 2$, and $RT = 64$, find the value of x. Then find RS and ST.

14. **Algebra** If $RS = 8y + 4$, $ST = 4y + 8$, and $RT = 15y - 9$, find the value of y. Then find RS and ST.

Use the figure at the right for Exercises 15–17.



15. If $m\angle MQV = 90$ and $m\angle VQP = 35$, what is $m\angle MQP$?

16. If $m\angle MVQ = 55$, what is $m\angle QVP$?

17. Judging by appearance, name each of the following.
- **a.** two acute angles
- **b.** two obtuse angles
- **c.** two right angles

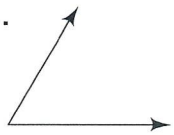
18. Without using your protractor, sketch angles with the following measures. Then use your protractor to see how close you are.
- **a.** 30 **b.** 60 **c.** 120

19. **Open-ended** The mileage markers on highways are an example of a numbering system that resembles a number line. Give another example.

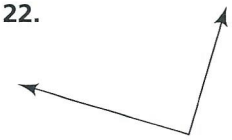
20. **Ski Jumping** This ski jumper is using a new style of jumping. The skis are at an angle rather than parallel. Measure the angle formed by the two skis.

Estimation **Estimate the measure of each angle. Then use a protractor to measure it. Classify each angle.**

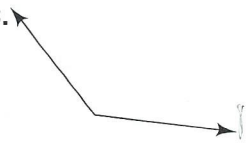
21.



22.



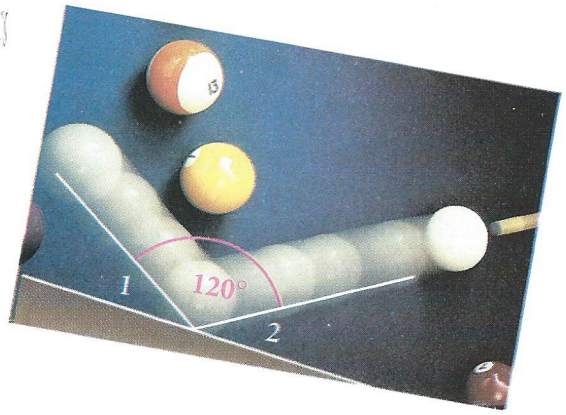
23.



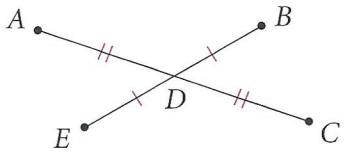
24. **Coordinate Geometry** $AB = 12$. Point A has coordinates $(3, 0)$. Give four possible locations for point B.

25. **Open-ended** Name two times when the hands of a clock
- **a.** form an acute angle. **b.** form a right angle.
- **c.** form an obtuse angle. **d.** form a straight angle.

26. **Billiards** In billiards, the cue ball may bounce off a cushion on any shot. If there is no spin on the shot, $\angle 1$ and $\angle 2$ will be congruent. Find the measures of $\angle 1$ and $\angle 2$.



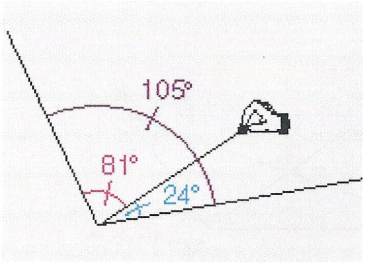
27. Algebra If $AD = 12$ and $AC = 4y - 36$, find the value of y.

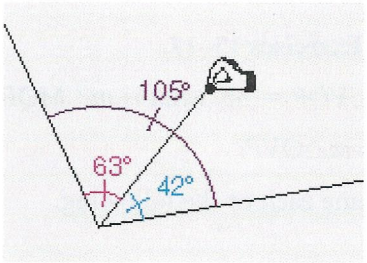


28. Algebra If $ED = x + 4$ and $DB = 3x - 8$, find EB.

29. Writing The word "acute" can mean *sharp* in conversational English. Explain why this meaning describes an acute angle.

30. Technology Leon constructed an angle. Then he constructed a ray from the vertex of the angle to a point in the interior of the angle. He measured all the angles formed. Then he moved the interior ray.

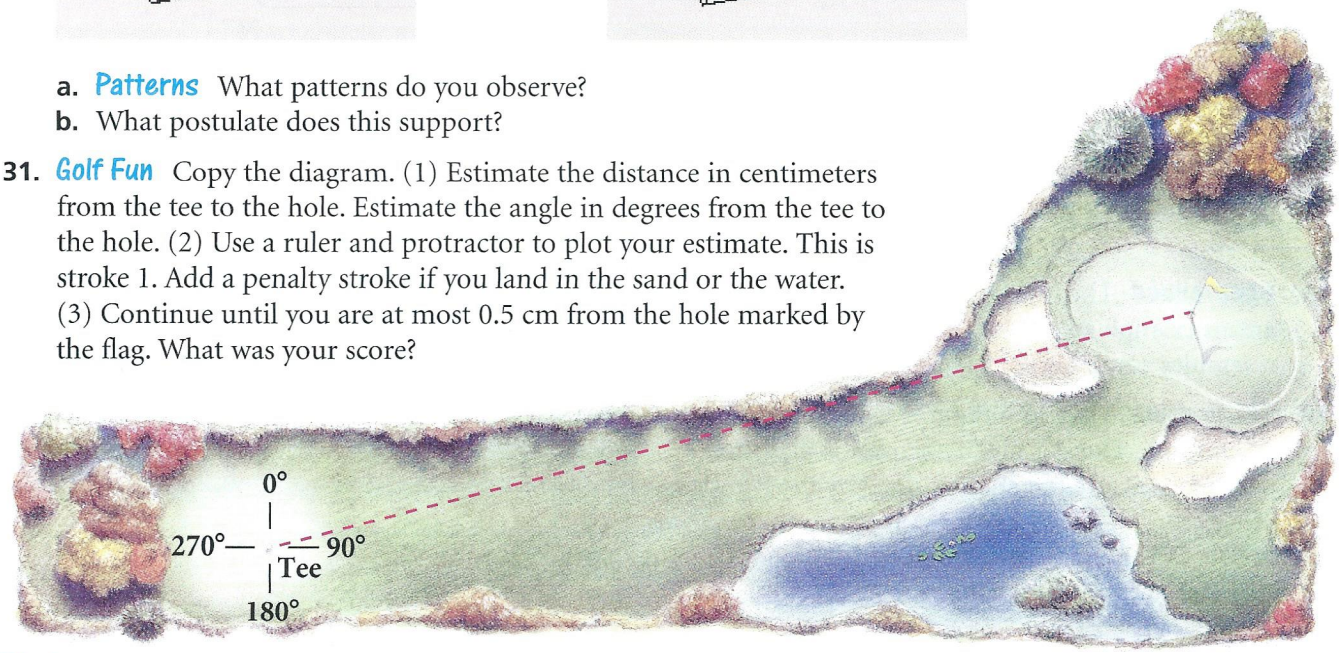




a. Patterns What patterns do you observe?

b. What postulate does this support?

31. Golf Fun Copy the diagram. (1) Estimate the distance in centimeters from the tee to the hole. Estimate the angle in degrees from the tee to the hole. (2) Use a ruler and protractor to plot your estimate. This is stroke 1. Add a penalty stroke if you land in the sand or the water. (3) Continue until you are at most 0.5 cm from the hole marked by the flag. What was your score?



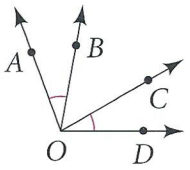
Algebra **Solve for x.**

32. $m\angle AOC = 7x - 2$, $m\angle AOB = 2x + 8$, $m\angle BOC = 3x + 14$

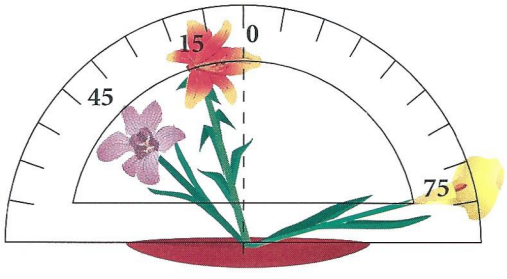
33. $m\angle AOB = 4x - 2$, $m\angle BOC = 5x + 10$, $m\angle COD = 2x + 14$

34. $m\angle AOB = 28$, $m\angle BOC = 3x - 2$, $m\angle AOD = 6x$

35. $m\angle AOB = 4x + 3$, $m\angle BOC = 7x$, $m\angle AOD = 16x - 1$



36. Decorating Japanese flower arranging makes precise use of angles to create a mood. A vertical stem is matched with 0. Other stems are matched with numbers from 0 to 90 in both directions from the vertical. What numbers would the flowers shown be paired with on a standard protractor?



Chapter Project **Find Out by Researching**

Choose one of the origami creations pictured on pages 2 and 3. Find a book that contains directions for making the figure, and follow the directions to make your favorite origami creation.

Exercises MIXED REVIEW

Find the next term in each sequence.

37. 5, 10, 15, 20, . . .

38. 5, 25, 125, 625, . . .

39. 14, 18, 22, 26, . . .

Complete each statement.

40. Three points are always __?__.

41. Intersecting lines are always __?__.

42. Two intersecting planes intersect in a __?__.

43. Two points are always __?__.

44. A 24-cm segment is divided into two segments. One segment is three times as long as the other. Find the lengths of both segments.

Getting Ready for Lesson 1-5

Sketch each figure.

45. $\overline{CD}$

46. $\overrightarrow{GH}$

47. $\overleftrightarrow{AB}$

48. line m

Exercises CHECKPOINT

Find the next two terms in each sequence.

1. 19, 21.5, 24, 26.5, . . .

2. 3.4, 3.45, 3.456, 3.4567, . . .

3. $-2, 6, -18, 54, \ldots$

4. **Writing** Describe the pattern of the sequences in Exercises 1–3.

Are the following coplanar?

5. *A, E, F, B*

6. *F, C, B, H*

7. $\overleftrightarrow{DC}$, point E

8. *D, G, B*

9. $\overleftrightarrow{GC}$, $\overleftrightarrow{BC}$

10. What is the intersection of plane *EFGH* and $\overleftrightarrow{DH}$?

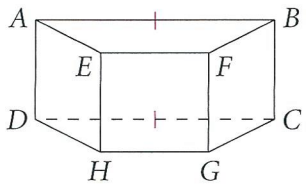
11. Name all the segments parallel to $\overline{HG}$.

12. Name two skew lines.

13. Name two parallel planes.

14. Name an acute, an obtuse, and a right angle.

15. **Algebra** If $AB = 4x + 5$ and $DC = 3x + 8$, find AB.



Exs. 5–15