4-2 Isosceles Triangles

What You'll Learn

- Using and applying properties of isosceles triangles

...And Why

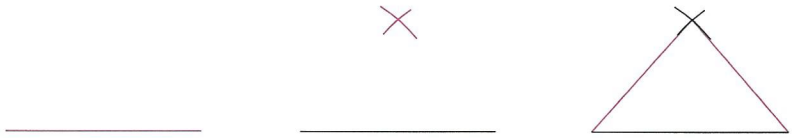
To understand a geometric figure used in the designs of many buildings and bridges

What You'll Need

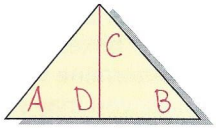
- straightedge
- compass
- scissors

WORK TOGETHER

Have each member of your group construct a different isosceles triangle and then cut it out. Be sure to include acute and obtuse triangles.



- Label the triangle $\triangle ABC$, with A and B opposite the congruent sides.
- Bisect $\angle C$ by folding the triangle so that the congruent sides overlap. Label the intersection of the fold line and $\overline{AB}$ as point D.

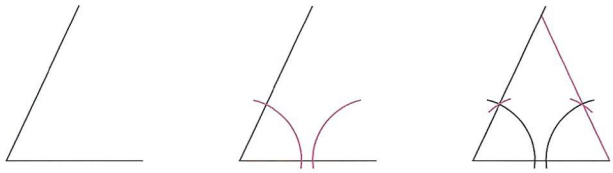


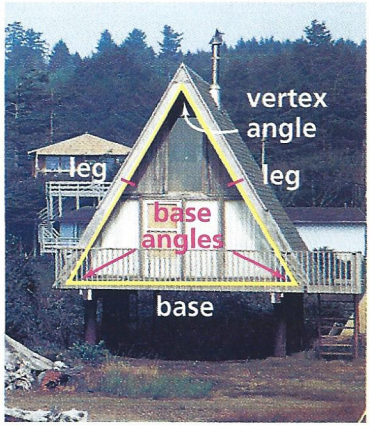
TECHNOLOGY HINT

The Work Together could be done using geometry software.

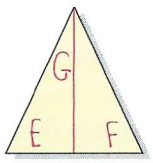
1. What do you notice about $\angle A$ and $\angle B$? Compare results within your group.
2. **a.** What types of angles do $\angle CDA$ and $\angle CDB$ appear to be?
   b. What do you notice about $\overline{AD}$ and $\overline{DB}$?
   c. Use your answers to parts (a) and (b) to complete the statement: $\overline{CD}$ is the __?__ of $\overline{AB}$.

- Construct a new triangle that has two congruent angles. Cut out the triangle and label it $\triangle EFG$, where $\angle E \cong \angle F$.





3. Fold the triangle so that the congruent angles overlap.
   a. What do you notice about $\overline{EG}$ and $\overline{FG}$? Compare results within your group.
   b. What type of triangle is $\triangle EFG$?



THINK AND DISCUSS

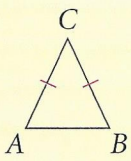
Isosceles triangles are common in the real world. You can find them in structures such as bridges and buildings. The congruent sides of an isosceles triangle are the **legs.** The third side is the **base.** The two congruent sides form the **vertex angle.** The other two angles are the **base angles.**

Your observations from the Work Together suggest the following theorems. The proofs of these theorems involve properties of congruent triangles that you will study in Chapter 8.

Theorem 4-1
Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are also congruent.

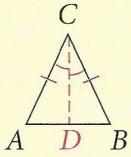
If $\overline{AC} \cong \overline{BC}$, then $\angle A \cong \angle B$.



Theorem 4-2

The bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base.

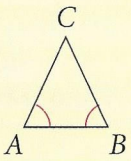
If $\overline{AC} \cong \overline{BC}$ and $\overline{CD}$ bisects $\angle ACB$, then $\overline{CD} \perp \overline{AB}$ and $\overline{CD}$ bisects $\overline{AB}$.



Theorem 4-3
Converse of Isosceles Triangle Theorem

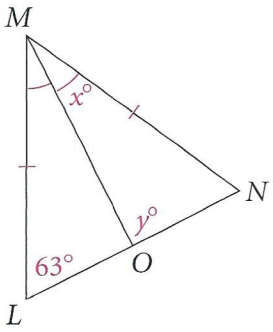
If two angles of a triangle are congruent, then the sides opposite the angles are congruent.

If $\angle A \cong \angle B$, then $\overline{AC} \cong \overline{BC}$.



4. Write the Isosceles Triangle Theorem and its converse as a biconditional.

Example 1



Algebra Find the values of x and y.

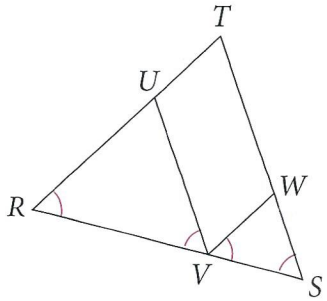
By Theorem 4-2, you know that $\overline{MO} \perp \overline{LN}$. So $y = 90$. Because the triangle is isosceles, $\angle L \cong \angle N$. So $m\angle N = 63$.

$m\angle N + x + y = 180$	**Triangle Angle-Sum Theorem**
$63 + x + 90 = 180$	**Substitution**
$x = 27$	**Subtract 153 from each side.**

So $x = 27$ and $y = 90$.

5. Try This Suppose $m\angle L = 43$. Find the values of x and y.

Example 2



Complete each statement. Explain your answers.

a. $\overline{RT} \cong$? **b.** $\overline{RU} \cong$? **c.** $\overline{VW} \cong$?

a. $\overline{RT} \cong \overline{ST}$ because $\angle R \cong \angle S$.

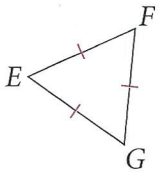
b. $\overline{RU} \cong \overline{VU}$ because $\angle R \cong \angle RVU$.

c. $\overline{VW} \cong \overline{SW}$ because $\angle WVS \cong \angle S$.

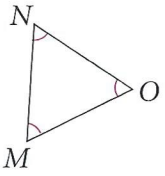
QUICK REVIEW

An equilateral triangle has three congruent sides. An equiangular triangle has three congruent angles.

6. a. Choose two sides of $\triangle EFG$. What must be true about the angles opposite these sides? Why?
b. Repeat part (a) with a different pair of sides.
c. What is true about the angles of an equilateral triangle?



7. a. Choose two angles of $\triangle MNO$. What must be true about the sides opposite these angles? Why?
b. Repeat part (a) with a different pair of angles.
c. What is true about the sides of an equiangular triangle?

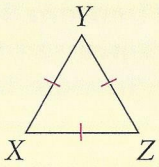


Your observations from Questions 6 and 7 are summarized below.

Corollary to Isosceles Triangle Theorem

If a triangle is equilateral, then it is equiangular.

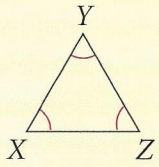
If $\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$, then $\angle X \cong \angle Y \cong \angle Z$.



Corollary to Converse of Isosceles Triangle Theorem

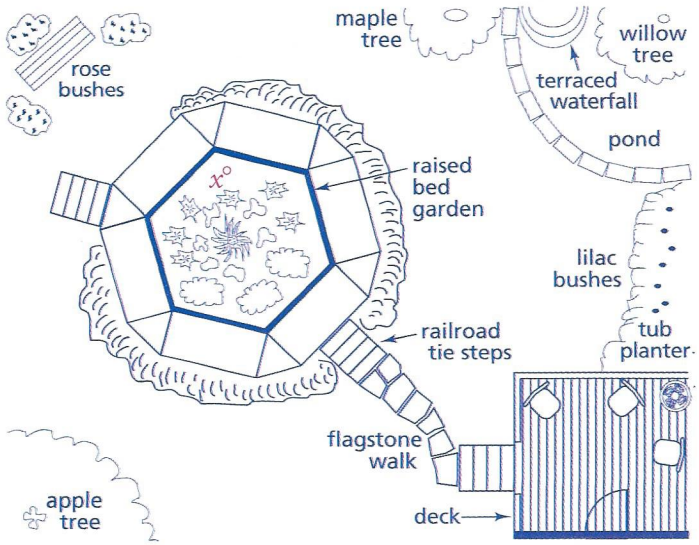
If a triangle is equiangular, then it is equilateral.

If $\angle X \cong \angle Y \cong \angle Z$, then $\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$.



8. Use the corollaries above to write a biconditional.

Example 3



Relating to the Real World

Landscaping A landscaper is building a raised bed garden to fit in the hexagonal space in the diagram. The path around the garden consists of rectangles and equilateral triangles. What is the measure of the angle marked x?

Each angle of a rectangle measures 90. Each angle of an equilateral triangle measures 60. (Why?)

$$x + 90 + 60 + 90 = 360$$
$$x = 120$$

The measure of the angle is 120.

Exercises ON YOUR OWN

Algebra **Find the values of x and y.**

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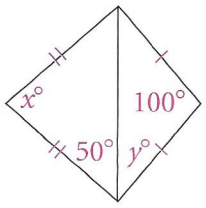

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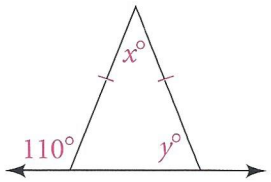

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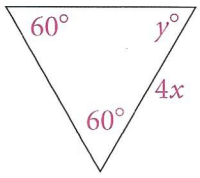
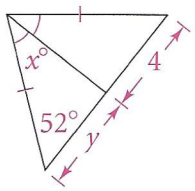


Perimeter is 54.

4.



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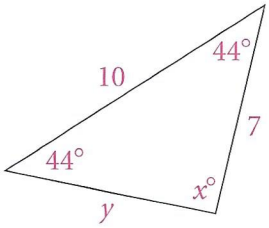

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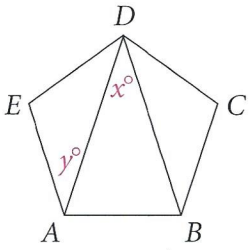
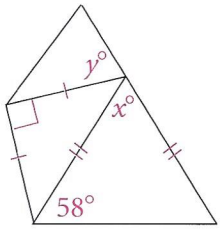


ABCDE is a regular pentagon.

7.



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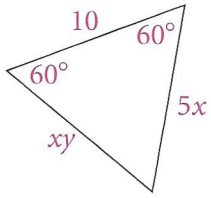
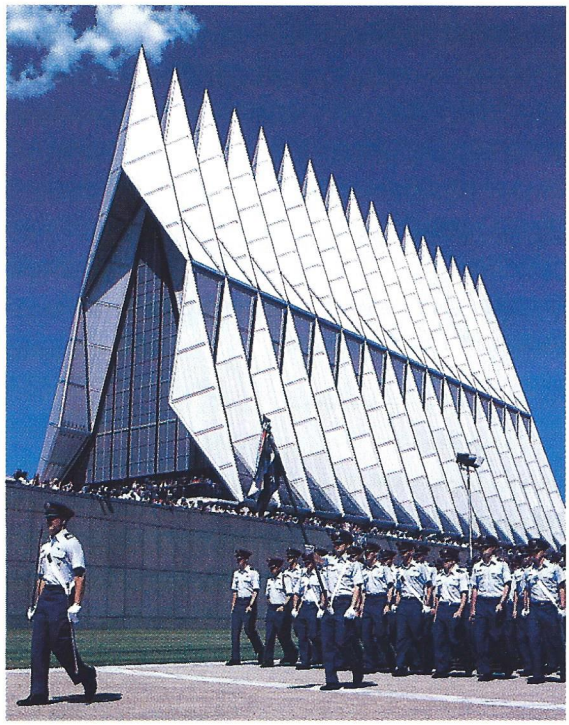


9. Architecture The Air Force Academy Cadet Chapel has 17 spires that point to the sky. Each spire is an isosceles triangle with a 40° vertex angle. Find the measures of the base angles.

10. Critical Thinking What are the measures of the base angles of an isosceles right triangle? Explain.

11. Coordinate Geometry The vertices of the base angles of an isosceles triangle are at (0, 0) and (6, 0). Describe the possible locations of the third vertex.



Logical Reasoning **Determine whether each statement is true or false. If it is false, provide a counterexample.**

12. If a quadrilateral is equilateral, then it is equiangular.

13. If a quadrilateral is equiangular, then it is equilateral.

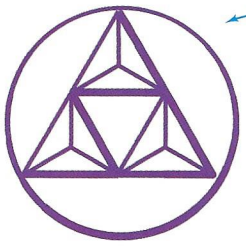
14. Every isosceles triangle has at least one line of symmetry.

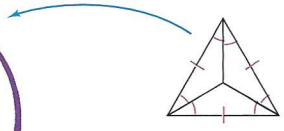
15. Every equilateral triangle has exactly three lines of symmetry.

16. Graphic Arts The logo of the National Council of Teachers of Mathematics is shown at the right.

a. Trace the logo onto your paper. Highlight an obtuse isosceles triangle in the design and then find its angle measures.

b. Open-ended Repeat part (a) for each of the following figures: kite, pentagon, hexagon.





The triangles in the logo have these congruent sides and angles.

Coordinate Geometry **For each pair of points, there are six points that could be the third vertex of an isosceles right triangle. Find the coordinates of each point.**

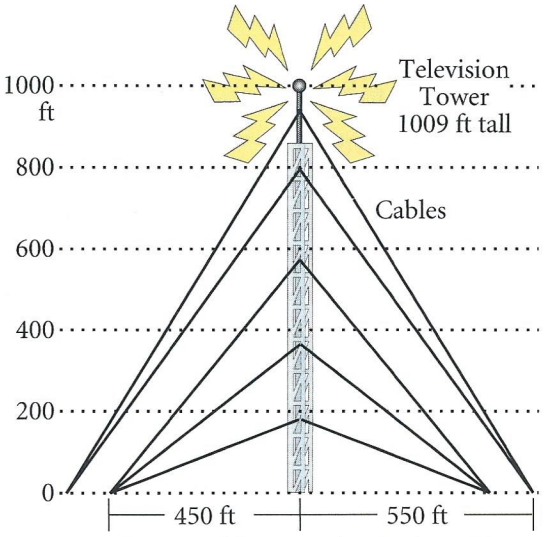
17. (0, 0) and (5, 5)

18. (2, 3) and (5, 6)

19. **Algebra** A triangle has angle measures $x + 15$, $3x - 35$, and $4x$.
- **a.** Find the value of x.
- **b.** Find the measure of each angle.
- **c.** What type of triangle is it? Why?

20. **Writing** If a triangle is equiangular, is it also isosceles? Explain.

21. **a.** **Communications** In the diagram, what type of triangle is formed by the cable pairs and the ground?
- **b.** What are the two different base lengths of the triangles?
- **c.** How is the tower related to each of the triangles?



Tower cables extend to both widths.

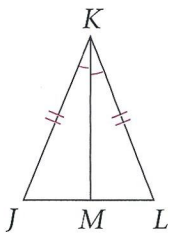
Find each value.

22. If $m\angle L = 58$, then $m\angle LKJ =$ ■.

23. If $JL = 5$, then $ML =$ ■.

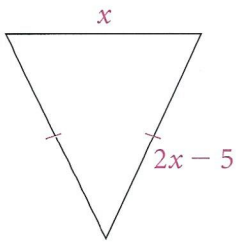
24. If $m\angle JKM = 48$, then $m\angle J =$ ■.

25. If $m\angle J = 55$, then $m\angle JKM =$ ■.



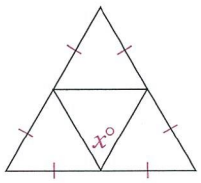
Choose **Use mental math, pencil and paper, or a calculator to find the values of the variables.**

26.



Perimeter is 20.

27.



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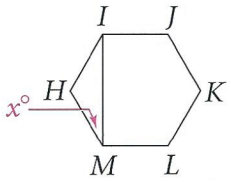
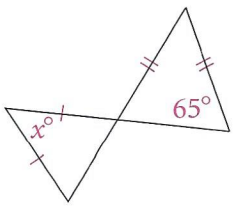


HIJKLM is a regular hexagon.

29.



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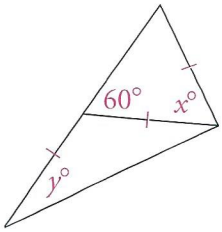

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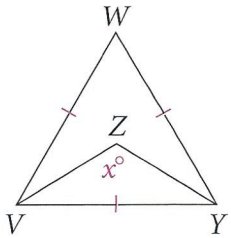
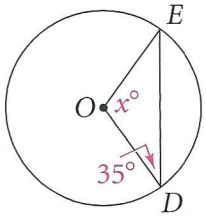


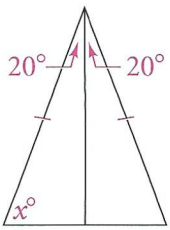
$\overline{VZ}$ and $\overline{YZ}$ are angle bisectors.

32.

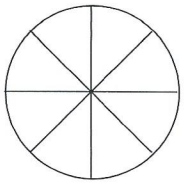


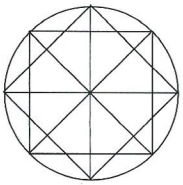
$\overline{OD}$ and $\overline{OE}$ are radii.

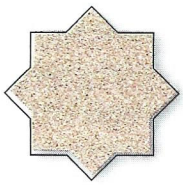
33.



34. Crafts This design is used in Hmong crafts and in Islamic and Mexican tiles. To create it, the artist starts by drawing a circle and four equally spaced diameters.


Step 1


Step 2

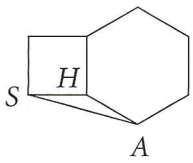

Step 3

a. How many different sizes of isosceles right triangles can you find in Step 2? Trace an example of each onto your paper.

b. For each size of triangle that you traced, count the number of times it appears in the diagram.

35. Critical Thinking Patrick defines the base of an isosceles triangle as "the bottom side of an isosceles triangle." Is his definition a good one? Explain why or why not.

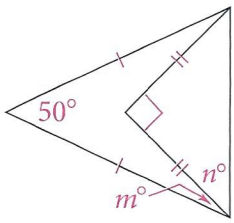
36. Standardized Test Prep A square and a regular hexagon are placed so that they have a common side. Find $m\angle HAS$.



A. 9 **B.** 10 **C.** 15 **D.** 20 **E.** 30

Algebra **Find the values of m and n.**

37.



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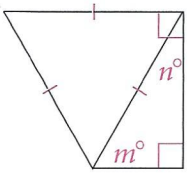

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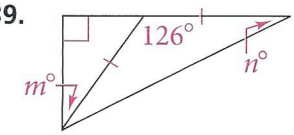
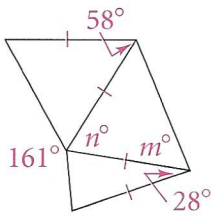


40.



Exercises MIXED REVIEW

Coordinate Geometry **The endpoints of a diameter of a circle are given. Find the coordinates of the center and the length of a radius.**

41. (3, 8), (−1, 2)

42. (−2, 5), (−5, 2)

43. (3, 7), (−2, 6)

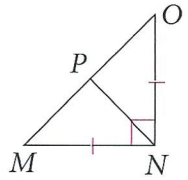
44. Coordinate Geometry Find the equation of the line that passes through (0, 4) and is parallel to $y = -3x - 5$.

45. Find the number of sides of a regular polygon whose exterior angles measure 15°.

Getting Ready for Lesson 4-3

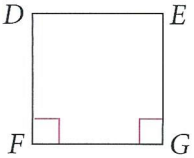
What can you conclude from each diagram? Justify your answers.

46.



$\angle MNP \cong \angle ONP$

47.



48.