

## What You'll Learn

- Writing and interpreting different types of conditional statements

## ...And Why

To help you analyze situations and become a better problem solver

4-1

# Using Logical Reasoning

## THINK AND DISCUSS

### Conditionals and Converses

- If Raúl's major is bagpipe, then he attends Carnegie-Mellon University.
  - If a second goes by, then Earth has moved another  $18\frac{1}{2}$  mi along its orbit.
  - If a movie is scary, then the concession stands sell more popcorn.
  - If you are not completely satisfied, then your money will be refunded.
1. You make and hear *if-then* statements like these many times each day. What are some *if-then* statements you have heard?

Another name for an *if-then* statement is a **conditional**. Every conditional has two parts. The part following *if* is the **hypothesis**, and the part following *then* is the **conclusion**.

### Example 1

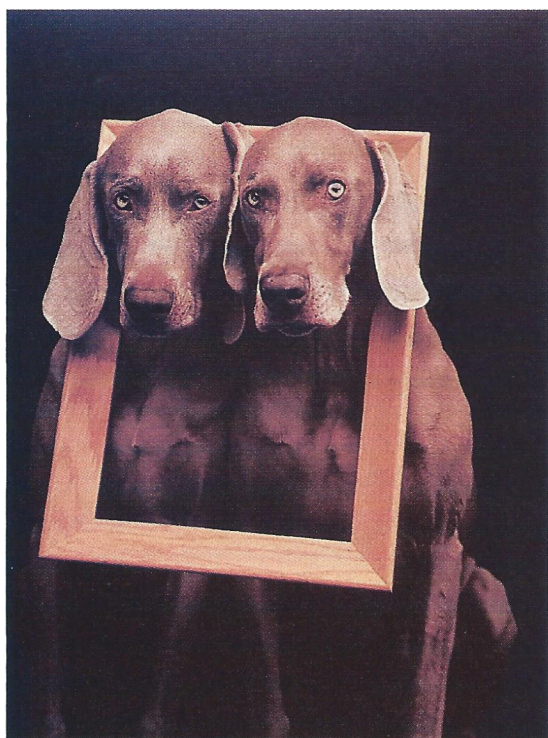
Identify the hypothesis and conclusion in this statement:

If it is February, then there are only 28 days in the month.

Hypothesis: It is February.

Conclusion: There are only 28 days in the month.

2. **Try This** Identify the hypothesis and conclusion in the photo caption at the left.



If you want double the love,  
then buy a pair.

When you determine whether a conditional is true or false, you determine its **truth value**. To show that a conditional is false, you need to find only one *counterexample* for which the hypothesis is true and the conclusion is false. The conditional in Example 1 is false because during a leap year February has 29 days.

3. Find a counterexample for this conditional: If the name of a state contains the word *New*, then the state borders an ocean.

Many sentences can be written as conditionals. For example, the sentence

Quadrilaterals have four sides.

can be rewritten in *if-then* form as

If a polygon is a quadrilateral, then it has four sides.

The **converse** of a conditional interchanges the hypothesis and conclusion.

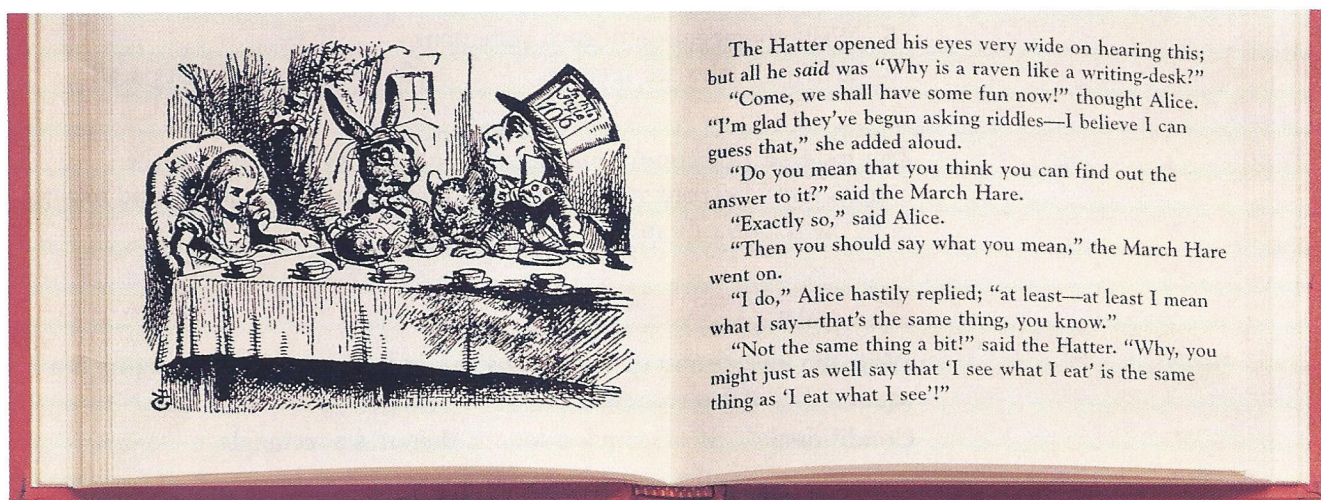
### Example 2

Write the converse of this statement: If a polygon is a quadrilateral, then it has four sides.

Conditional: If a polygon is a quadrilateral, then it has four sides.

Converse: If a polygon has four sides, then it is a quadrilateral.

**Literature** Notice that both statements in Example 2 have the same truth value. This is *not* true of all conditionals and their converses, as Alice discovers in this passage from Lewis Carroll's *Alice's Adventures in Wonderland*.



The Hatter's statement "I see what I eat" can be rewritten in if-then form as "If I eat it, then I see it." The converse, "I eat what I see," can be rewritten as "If I see it, then I eat it."

4. Are the truth values for the Hatter's statement and its converse the same? Explain.

### Biconditionals, Inverses, and Contrapositives

When a conditional and its converse are true, you can combine them as a **biconditional**. The conditionals from Example 2 can be combined as

If a polygon is a quadrilateral, then it has four sides, *and* if a polygon has four sides, then it is a quadrilateral.

This long sentence can be shortened by using the phrase *if and only if*:

A polygon is a quadrilateral *if and only if* it has four sides.

You learned in Chapter 1 that any good definition is “reversible.” This means that any good definition can be written as a biconditional.

### Example 3

Write the two statements that make up this definition: A right angle has measure 90. Then write the definition as a biconditional.

Conditional: If an angle is a right angle, then its measure is 90.

Converse: If the measure of an angle is 90, then it is a right angle.

Biconditional: An angle is a right angle if and only if its measure is 90.

The **negation** of a statement has the opposite meaning. For example, the negation of “An angle is acute” is “An angle is *not* acute.”

5. Write the negation of each statement.
  - a. Two angles are vertical.
  - b. Two lines are not parallel.

The **inverse** of a conditional negates both the hypothesis and the conclusion. The **contrapositive** of a conditional interchanges and negates both the hypothesis and the conclusion.

### Example 4

Write the inverse and the contrapositive of this statement: If a figure is a square, then it is a rectangle.

Conditional: If a figure is a square, then it is a rectangle.

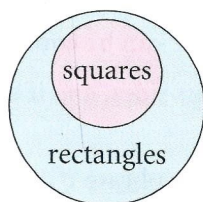
Negate both.

Inverse: If a figure is *not* a square, then it is *not* a rectangle.

Conditional: If a figure is a square, then it is a rectangle.

Negate both.

Contrapositive: If a figure is *not* a rectangle, then it is *not* a square.



6. Find the truth values of the three statements in Example 4.
7. **Try This** Write the inverse and contrapositive of this statement: If a quadrilateral is a parallelogram, then it has two pairs of parallel sides.

This Venn diagram shows the conditional “If a figure is a square, then it is a rectangle.” It also shows its contrapositive “If a figure is *not* a rectangle, then it is *not* a square” because any point *not* in the larger circle is *not* in the smaller circle. Since the same diagram shows both statements, a conditional and its contrapositive always have the same truth value.

## Summary of Conditionals

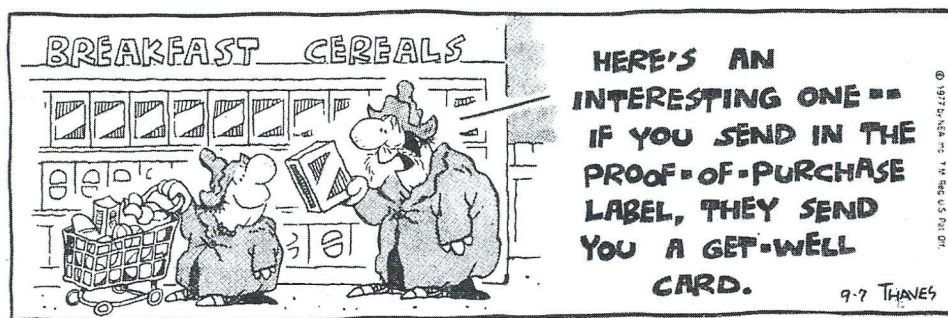
Statement	Form	Example
conditional	If ■, then ■.	If an angle is a straight angle, then its measure is 180.
converse	If ■, then ■.	If the measure of an angle is 180, then it is a straight angle.
inverse	If not ■, then not ■.	If an angle is not a straight angle, then its measure is not 180.
contrapositive	If not ■, then not ■.	If the measure of an angle is not 180, then it is not a straight angle.
biconditional	■ if and only if ■.	An angle is a straight angle if and only if its measure is 180.

## Exercises ON YOUR OWN

1. Identify the hypothesis and conclusion in the cartoon.

FRANK AND ERNEST

By BOB THAVES



- For Exercises 2–5: (a) Rewrite each statement in if-then form.  
 (b) Underline the hypothesis once and the conclusion twice.

2. Glass objects are fragile.
3.  $3x - 7 = 14$  implies that  $3x = 21$ .
4. Numbers that have 2 as a factor are even.
5. An isosceles triangle has two congruent sides.

Find a counterexample for each statement.

6. If it is not a weekday, then it is Saturday.
7. Odd integers less than 10 are prime.
8. If you live in a country that borders the United States, then you live in Canada.
9. If you play a sport with a ball and bat, then you play baseball.
10. a. **Open-ended** Write a conditional with the same truth value as its converse.  
 b. Write a conditional whose converse has the opposite truth value.

For each statement, write (a) the converse, (b) the inverse, and (c) the contrapositive.

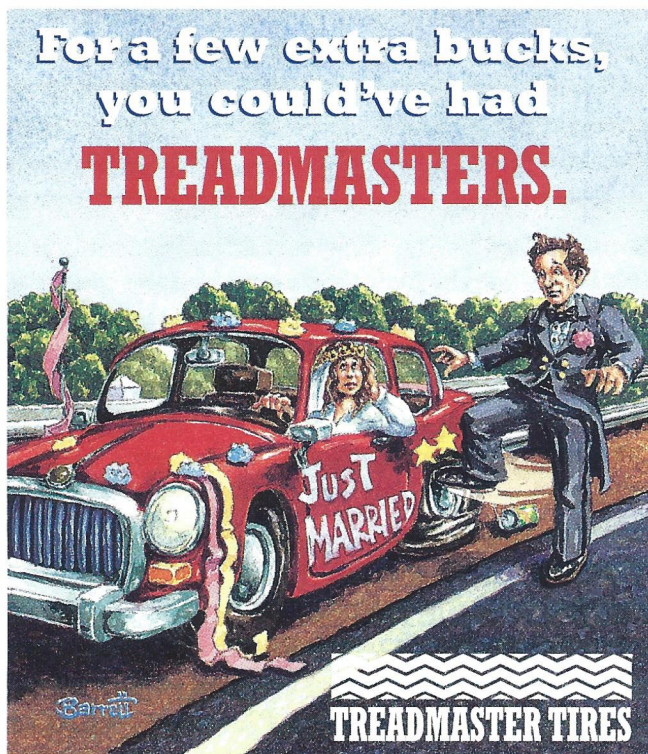
11. If you eat all of your vegetables, then you will grow.
13. If a triangle is a right triangle, then it has a  $90^\circ$  angle.
15. If two segments are congruent, then they have the same length.
17. If a polygon is a pentagon, then the sum of the measures of its angles is 540.
12. **Transformations** If a figure has point symmetry, then it has rotational symmetry.
14. If a quadrilateral has exactly two congruent sides, then it is not a rhombus.
16. If you do not work, you will not get paid.
18. If a conditional statement is false, then its contrapositive is false.

For Exercises 19–26: (a) Write the converse of each statement.  
(b) Determine the truth value of the statement and its converse.  
(c) If both statements are true, write a biconditional.

19. If you travel from the United States to Kenya, then you have a passport.
21. **Chemistry** If a substance is water, then its chemical formula is  $H_2O$ .
23. **Coordinate Geometry** If two nonvertical lines are parallel, then their slopes are equal.
25. If you are in Indiana, then you are in Indianapolis.
27. a. **Consumer Issues** Advertisements often suggest conditional statements. For example, an ad might imply that if you don't buy a product, you won't be popular. What conditional is implied in the ad at the right?  
b. **Research** Find magazine ads that use conditionals effectively. Make a poster to display these ads.
20. **Coordinate Geometry** If a point is in the first quadrant, then its coordinates are positive.
22. **Transformations** If a figure has two lines of symmetry, then it has rotational symmetry.
24. If two angles are complementary, then the sum of their measures is 90.
26. **Probability** If the probability that an event will occur is 1, then the event is certain to occur.

Write the two conditionals that make up each biconditional.

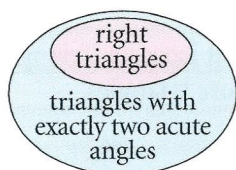
28. A swimmer wins a race if and only if she swims the fastest.
29. A number is divisible by 3 if and only if the sum of its digits is divisible by 3.
30. Two angles are congruent if and only if they have the same measure.



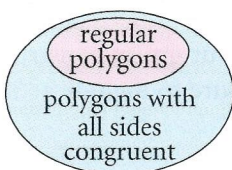
31. **Writing** Jynona knows that vertical angles are congruent. She thinks the converse is also true. Is she correct? Explain.

For each Venn diagram: (a) Write a conditional statement. (b) Write the contrapositive of the conditional.

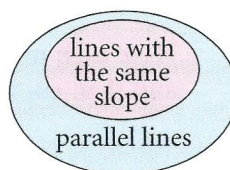
32.



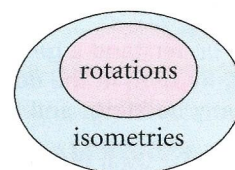
33.



34.



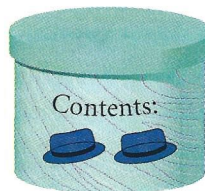
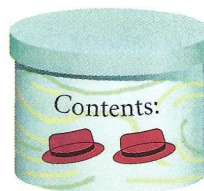
35.



## Chapter Project

### Find Out by Listing

Three red hats and three blue hats are packed in three boxes, with two hats to a box. The boxes are all labeled incorrectly. To determine what each box actually contains, you may select one hat from one box, without looking at the contents of that box. Explain how this will allow you to determine the contents of each box. (*Hint: List all possible solutions; then use logic to solve.*)



## Exercises MIXED REVIEW

**Coordinate Geometry** Sketch a line with the given slope containing the given point.

36.  $m = \frac{1}{2}, (2, -6)$

37.  $m = 1, (0, 5)$

38.  $m = -2, (-3, 6)$

39.  $m = \frac{1}{3}, (0, 0)$

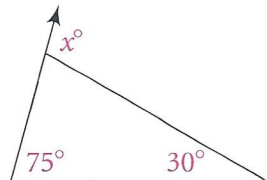
40. An angle's measure is 10 more than its supplement. Find the measures of both angles.

41. **Transformational Geometry** Locate the coordinates of the image of  $\triangle ABC$  with vertices  $A(0, 3)$ ,  $B(-4, -6)$ ,  $C(6, 1)$  under a  $180^\circ$  rotation about the origin.

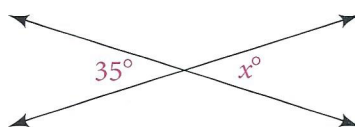
### Getting Ready for Lesson 4-2

Find the value of  $x$ .

42.



43.



44.

