3-3 Rotations

What You'll Learn

- Identifying and locating rotation images of figures

...And Why

To understand real-life objects that involve rotation, such as clocks, combination locks, and laser disc players

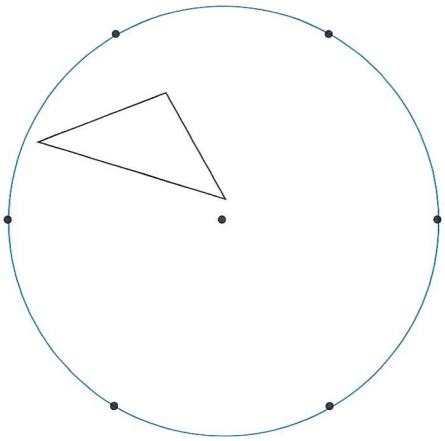
What You'll Need

- straightedge
- colored pencils (optional)
- protractor
- compass

WORK TOGETHER

Before beginning the activity, have each member of your group fold a piece of paper in half lengthwise and widthwise and then cut it into fourths.

Step 1: Place a piece of the paper over the figure below. Trace the six points on the circle, the center of the circle, and the triangle.



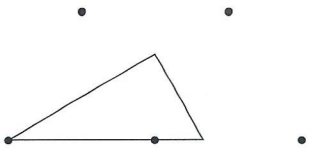
Step 2: Place the point of your pencil on the center of the circle and then rotate the paper until the six points again overlap. Trace the triangle in its new location.

Step 3: Repeat Step 2 until there are six triangles on your paper. Compare drawings within your group to be sure that your results look the same.

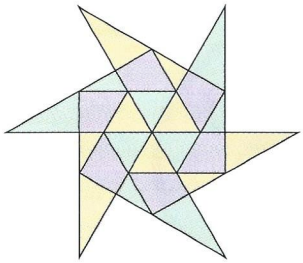
Step 4: Now it's your turn to be creative. Place a piece of paper over the figure above, trace the six points on the circle and the center of the circle, and then draw your own triangle on the paper.

Step 5: Place the paper from Step 4 on your desktop, and then use a blank piece of paper to repeat the process in Steps 1–3. Color your design, and then create a display of your group's designs.

If you start with this in Step 4 . . .



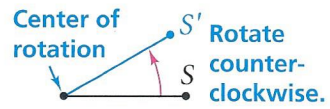
You could end up with this . . .

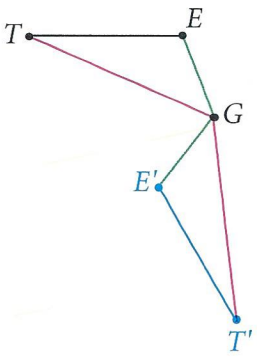


THINK AND DISCUSS

In the Work Together, you used rotations to create a design. In order to describe a rotation, you need to know the center of rotation, the angle of rotation, and the direction of the rotation.

The direction of a rotation can be clockwise or counterclockwise. All rotations in this book will be in a *counterclockwise* direction.

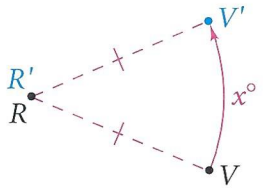




1. What was the angle of each rotation in the Work Together? (*Hint:* Each angle had the same measure.)
2. The diagram at the left shows $\overline{TE}$ rotated 120° about G.
   a. What appears to be true of EG and $E'G$? Of TG and $T'G$?
   b. What appears to be true of $m\angle TGT'$ and $m\angle EGE'$?

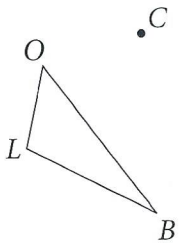
The properties of rotations that you noted in Question 2 form the basis of the definition of a rotation. A **rotation** of $x°$ about a point R is a transformation such that:

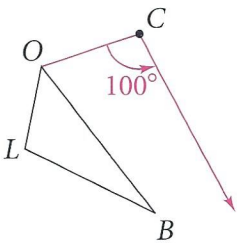
- For any point V, $RV' = RV$ and $m\angle VRV' = x$.
- The image of R is itself (that is, $R' = R$).



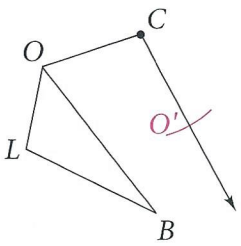
Example 1

Draw the image of $\triangle LOB$ under a 100° rotation about C.

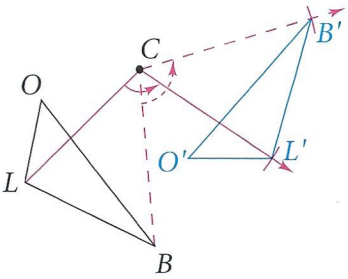




Step 1
Use a protractor to draw a 100° angle with side $\overrightarrow{OC}$.



Step 2
Use a compass to construct $\overline{O'C} \cong \overline{OC}$.



Step 3
Create B' and L' in a similar manner. Draw $\triangle L'O'B'$.

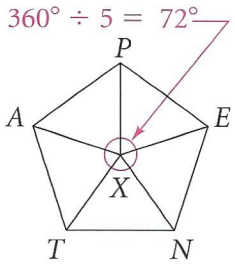
3. **Try This** Draw the image of $\triangle LOB$ under a 90° rotation about B.
4. **Critical Thinking** Under a rotation, does each point move the same distance? If not, which points move the farthest?

A comparison of $\triangle LOB$ and $\triangle L'O'B'$ in Example 1 reveals the following properties of a rotation.

Properties of a Rotation

A rotation is an isometry.

A rotation does not change orientation.



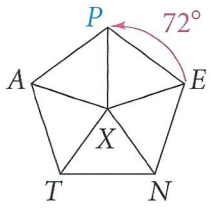
Example 2

Regular pentagon *PENTA* is divided into five congruent triangles.

a. Name the image of *E* under a 72° rotation about *X*.

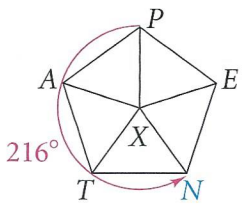
b. Name the image of *P* under a 216° rotation about *X*.

a.



P is the image of *E*.

b.



N is the image of *P*.

5. Try This Name the image of *T* under a 144° rotation about *X*.

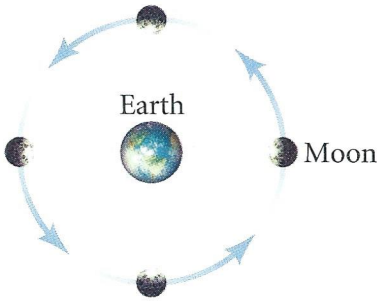
6. In Example 2, $E \longrightarrow P$, then $P \longrightarrow N$. Describe a single rotation that maps *E* to *N*.

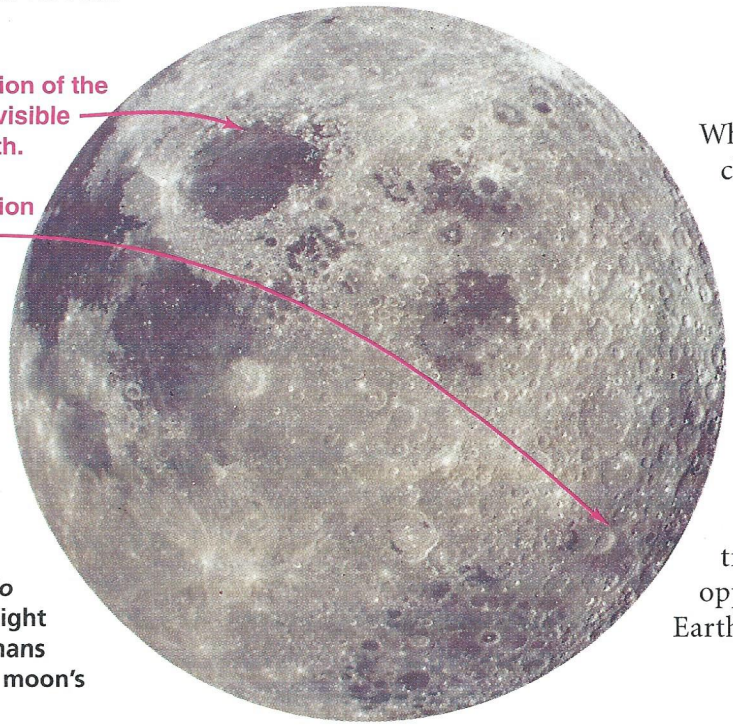
7. What is the image of any point under a 360° rotation?

What? Satellite photographs of the opposite side of the moon reveal that it is free of the large, dark regions that are visible from Earth. Early astronomers thought these regions looked like seas, so they named them *maria,* the Latin word for seas.

Example 3 Relating to the Real World

Astronomy The motion of the moon around Earth can be modeled by the type of rotation presented in this lesson. What effect does this have on our observations of the moon?





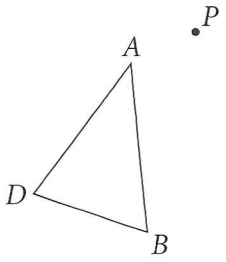
This photo was taken in 1972 by astronauts aboard *Apollo 17,* the last flight in which humans explored the moon's surface.

When a figure is rotated, the point closest to the center of rotation remains closest to the center of rotation. In the case of the moon, the point on its surface closest to Earth remains closest to Earth as it moves. This means that from Earth, only one side of the moon is ever visible. Our knowledge of the moon was "one-sided" until 1966, when lunar probes transmitted pictures of the opposite side of the moon back to Earth.

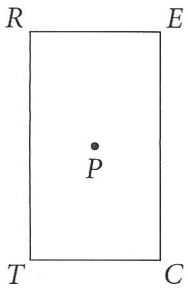
Exercises ON YOUR OWN

Copy each figure and point *P*. Rotate the figure the given number of degrees about *P*. Label the vertices of the image.

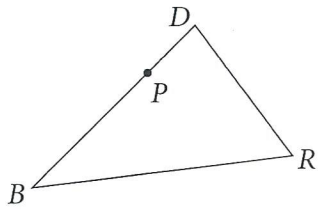
1. 60°



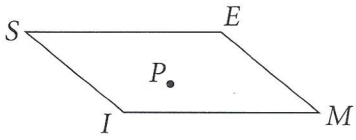
2. 90°



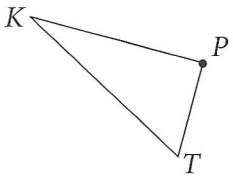
3. 180°



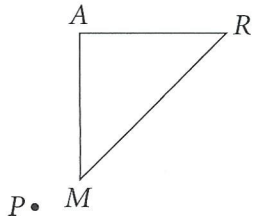
4. 140°



5. 90°

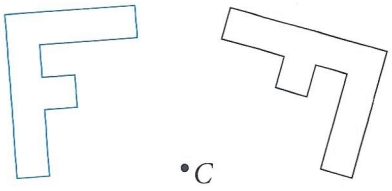


6. 45°

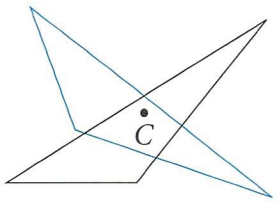


What is the measure of the rotation about *C* that maps the black figure onto the blue figure?

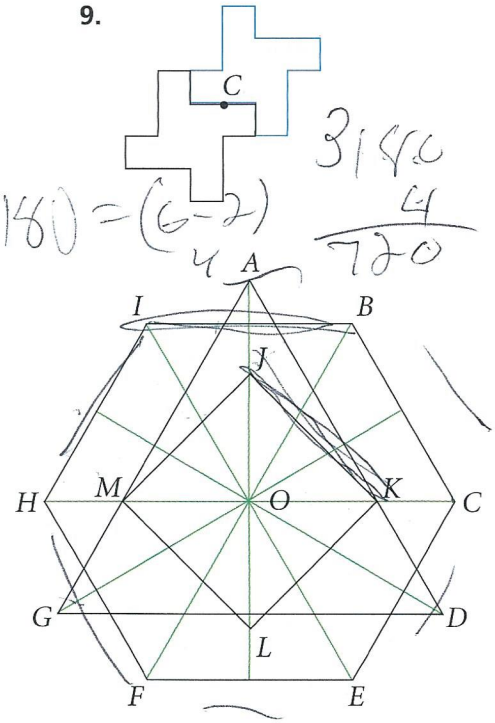
7.



8.



9.



The green segments in the figure intersect to form 30° angles. The triangle, quadrilateral, and hexagon are all regular. Find the image of each point or segment.

10. 120° rotation of *B* about *O*

11. 270° rotation of *L* about *O*

12. 60° rotation of *E* about *O*

13. 300° rotation of $\overline{IB}$ about *O*

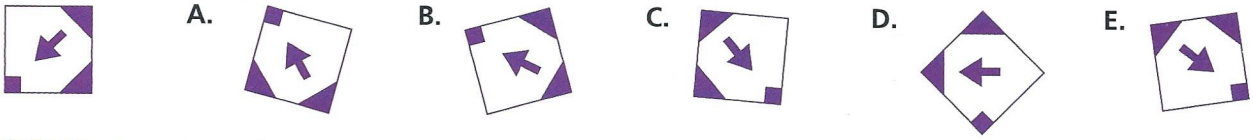
14. 120° rotation of $\overline{FE}$ about *O*

15. 120° rotation of *F* about *H*

16. 180° rotation of $\overline{JK}$ about *O*

17. 90° rotation of *L* about *M*

18. Standardized Test Prep Which figure is *not* the image of the figure at the left under a congruence transformation?



19. Astronomy Refer to Example 3. Suppose Earth's motion around the sun, like the moon's motion around Earth, could be described by the type of rotation in this lesson. How would life on Earth be different?

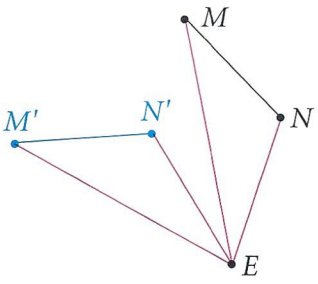
20. a. Coordinate Geometry Graph $A(5, 2)$, then graph B, the image of A under a 90° rotation about O (the origin). (*Hint:* Consider the slope of $\overline{OA}$.)

b. Graph C, the image of A under a 180° rotation about O.

c. Graph D, the image of A under a 270° rotation about O.

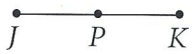
d. What type of quadrilateral is $ABCD$? Explain.

21. $\overline{M'N'}$ is the rotation image of $\overline{MN}$ about point E. Name all the congruent angles and segments in the diagram.

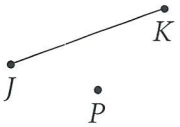


Copy each figure, then draw the image of $\overline{JK}$ under a 180° rotation about P.

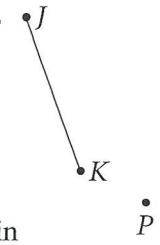
22.



23.



24.



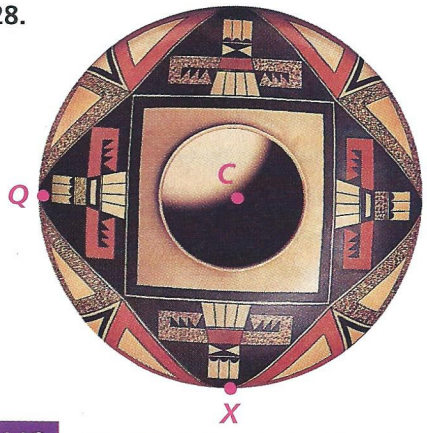
25.

26. Language Arts The symbol ə is called a *schwa*. It is used in dictionaries to represent neutral vowel sounds such as *a* in *ago*, *i* in *sanity*, and *u* in *focus*. What transformation maps a ə to a lowercase e?

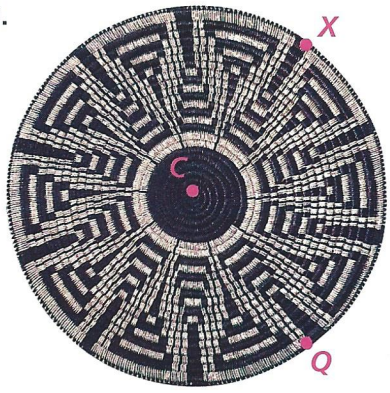
27. Open-ended Find a composition of rotations that has the same effect as a 360° rotation about a point X.

Native American Art **Find the measure of the rotation about C that maps Q to X.**

28.



29.



30.