3-2 Translations

What You'll Learn

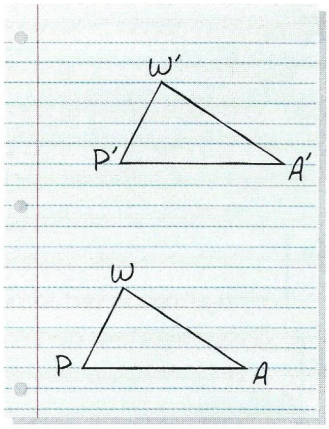
- Finding translation images of figures
- Using vectors and matrix addition to represent translations

...And Why

To use translations in the arts, computer graphics, navigation, manufacturing, music, and other fields

What You'll Need

- centimeter ruler
- scissors
- graph paper



WORK TOGETHER

- Have each member of your group draw a triangle, cut it out, and label it $\triangle PAW$.
- Place your triangle on a sheet of lined paper so that $\overline{PA}$ lies on a horizontal line. Trace the triangle, and label it $\triangle PAW$.
- Slide the cutout to another location on your paper so that $\overline{PA}$ again lies on a horizontal line. Trace the triangle, and label it $\triangle P'A'W'$.

1. Does the transformation $\triangle PAW \longrightarrow \triangle P'A'W'$ appear to be an isometry? Explain.
2. Does the transformation $\triangle PAW \longrightarrow \triangle P'A'W'$ change the orientation of the triangle? Explain.

- Use a straightedge to draw $\overline{PP'}$, $\overline{AA'}$, and $\overline{WW'}$. Measure each segment with a ruler.

3. What do you notice about the lengths of the segments?
4. Notice the positions of $\overline{PP'}$, $\overline{AA'}$, and $\overline{WW'}$ in relation to one another. What appears to be true about them? Compare your answer with others in your group.

THINK AND DISCUSS

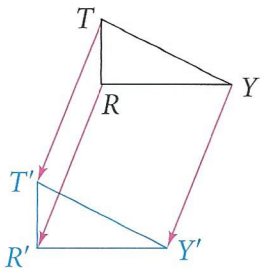
The sliding motion that maps $\triangle PAW$ to $\triangle P'A'W'$ in the Work Together is an example of a translation. A **translation** is a transformation that moves points the same distance and in the same direction. In the Work Together, you discovered the following properties of a translation.

Properties of a Translation

A translation is an isometry.

A translation does not change orientation.

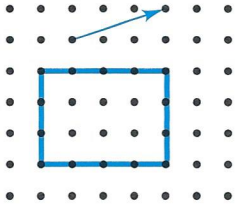
5. Elevators, escalators, and people movers all suggest translations. Name some other examples of translations from the real world.

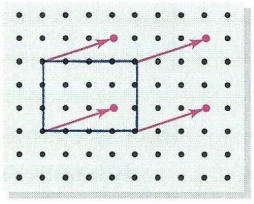


The distance and direction of a translation can be expressed as a *vector*. In the diagram, $\overrightarrow{TT'}$, $\overrightarrow{RR'}$, and $\overrightarrow{YY'}$ are vectors. Vectors have an *initial point* and a *terminal point*. T, R, and Y are initial points, and T', R', and Y' are terminal points. Note that although diagrams of vectors look identical to diagrams of rays, vectors do not go on forever in the indicated direction—they have a fixed length.

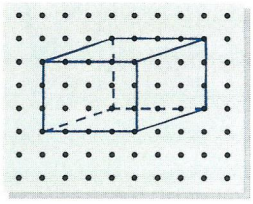
Example 1

Use the given vector and rectangle to create a sketch of a box.



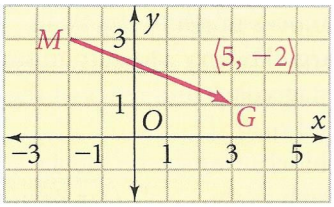


Step 1



Step 2

Copy the rectangle, then translate each of its vertices 3 units to the right and 1 unit up. Next, connect points to form the box. Use dashed lines for parts of the figure that are hidden from view.

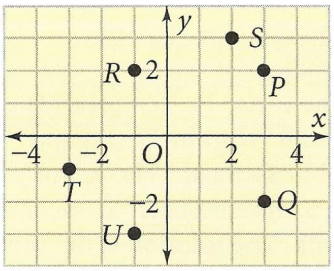


You can use *ordered pair notation*, $\langle x, y \rangle$, to represent a vector on the coordinate plane. In the notation, x represents horizontal change from the initial point to the terminal point and y represents vertical change from the initial point to the terminal point. The notation for vector $\overrightarrow{MG}$ is $\langle 5, -2 \rangle$.

6. Try This Describe the vector in Example 1 by using ordered pair notation.

7. Use vector notation to describe the vector with initial point (1, 3) and terminal point (6, 1).

Example 2



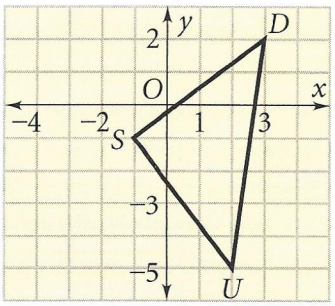
a. What is the image of P under the translation $\langle 0, -4 \rangle$?
b. What vector describes the translation $S \longrightarrow U$?

a. The vector $\langle 0, -4 \rangle$ represents a translation of 4 units down. The image of P is Q.
b. To get from S to U, you move 3 units left and 6 units down. The vector that describes this translation is $\langle -3, -6 \rangle$.

8. Try This Refer to the diagram in Example 2.
a. What is the image of S under the translation $\langle -3, -1 \rangle$?
b. What vector describes the translation $T \longrightarrow P$?

9. Describe in words the distance and direction of the translation represented by the vector $\langle 18, 0 \rangle$.

$$\begin{matrix} & S & U & D \\ x\text{-coordinate} \\ y\text{-coordinate} \end{matrix}\begin{bmatrix} -1 & 2 & 3 \\ -1 & -5 & 2 \end{bmatrix}$$



You can use matrices to help you translate figures in the coordinate plane. To do so, start by creating a matrix for the figure, as shown at the left.

Example 3

Use matrices to find the image of $\triangle SUD$ under the translation $\langle 4, -5 \rangle$.

To find the image of $\triangle SUD$, you add 4 to all of the x-coordinates and -5 to all of the y-coordinates.

Vertices of Preimage $\begin{matrix} S & U & D \end{matrix}$

$$\begin{bmatrix} -1 & 2 & 3 \\ -1 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 4 & 4 \\ -5 & -5 & -5 \end{bmatrix} = \begin{matrix} S' & U' & D' \end{matrix}\begin{bmatrix} 3 & 6 & 7 \\ -6 & -10 & -3 \end{bmatrix}$$

Translation Matrix

Vertices of Image

10. Check the answer to Example 3 by sketching $\triangle SUD$ and $\triangle S'U'D'$ on the same set of axes.

Example 4 Relating to the Real World

Travel Yolanda Pérez is visiting San Francisco. From her hotel near Union Square, she walked 4 blocks east and 4 blocks north to the Wells Fargo History Museum to see a stagecoach and relics of the gold rush. Then she walked 5 blocks west and 3 blocks north to the Cable Car Barn Museum. How many blocks from her hotel is she now?

As shown in the diagram, she is 1 block west and 7 blocks north of her hotel.

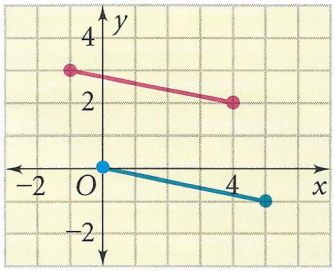
You can also solve this problem by using vectors. The vector $\langle 4, 4 \rangle$ represents a walk of 4 blocks east and 4 blocks north. The vector $\langle -5, 3 \rangle$ represents her second walk. The solution is the sum of the x- and y-coordinates of each vector:
$\langle 4, 4 \rangle + \langle -5, 3 \rangle = \langle -1, 7 \rangle$.

Example 4 shows the composition of two translations. The term **composition** describes any two transformations in which the second transformation is performed on the image of the first transformation. As the solution to Example 4 suggests, a composition of translations can be rewritten as a single translation.

Exercises ON YOUR OWN

In each diagram, the blue figure is the image of the red figure. Use ordered pair notation to represent each translation.

1.



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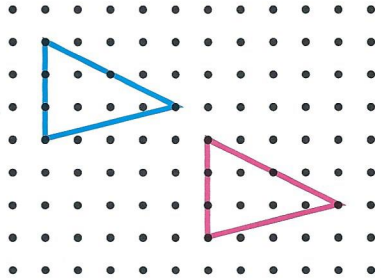

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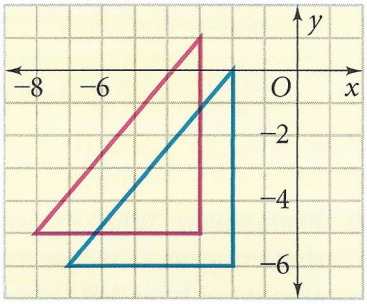

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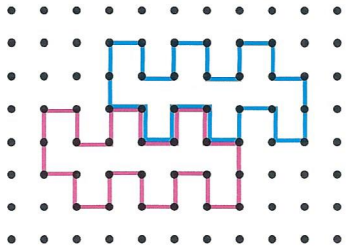

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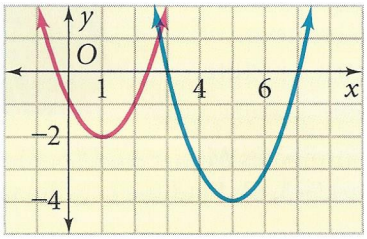

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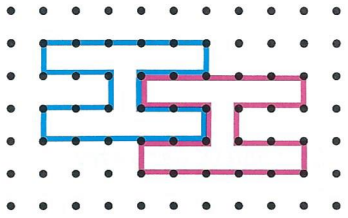
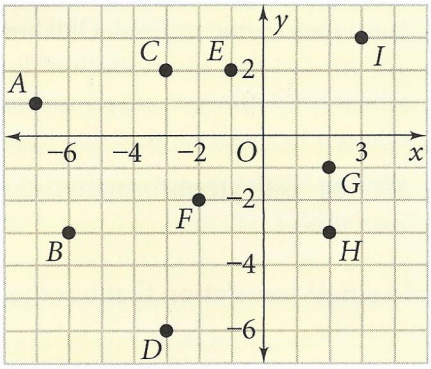


In Exercises 7–12, refer to the figure at the right.



7. What is the image of F under the translation $\langle -1, 4 \rangle$?

8. What vector describes the translation $G \longrightarrow H$?

9. What is the image of E under the translation $\langle 4, 1 \rangle$?

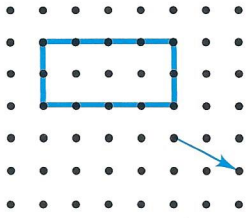
10. What vector describes the translation $B \longrightarrow E$?

11. What is the image of F under the translation $\langle 4, -1 \rangle$?

12. What vector describes the translation $I \longrightarrow C$?

Use each figure and vector to sketch a three-dimensional figure.

13.



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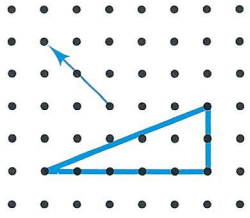
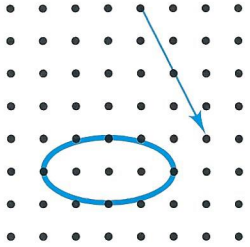
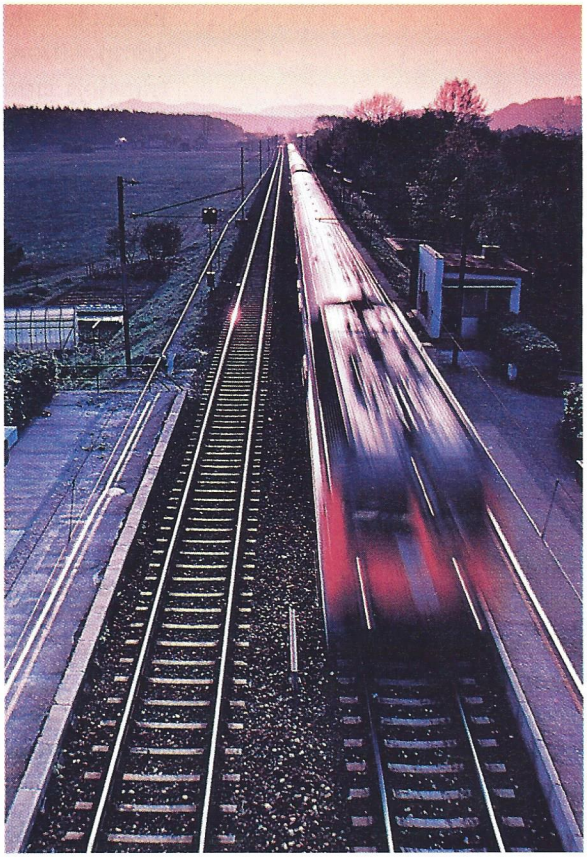


15.



16. Sailing Emily left Galveston Bay at the east jetty and sailed 4 km north to an oil rig. She then sailed 5 km west to Redfish Island. Finally, she sailed 3 km southwest to the Spinnaker Restaurant. Draw vectors on graph paper that show her journey.



In Exercises 17–19, use matrix addition to find the image of each figure under the given translation.

17. Figure: $\triangle ACE$ with vertices $A(7, 2)$, $C(-8, 5)$, $E(0, -6)$
Translation: $\langle -9, 4 \rangle$

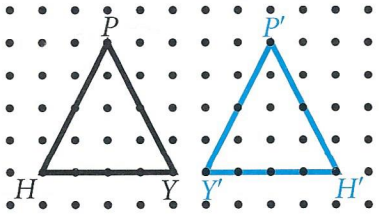
18. Figure: $\triangle PUN$ with vertices $P(1, 0)$, $U(4, 6)$, $N(-5, 8)$
Translation: $\langle 11, -13 \rangle$

19. Figure: $\square NILE$ with vertices $N(2, -5)$, $I(2, 2)$, $L(-3, 4)$, $E(-3, -3)$
Translation: $\langle -3, -4 \rangle$

20. Photography When you snap a photograph, a shutter opens to expose the film to light. The amount of time that the shutter remains open is known as the *shutter speed*. The photographer of the train used a long shutter speed to create an image that suggests a translation. Sketch a picture of your own that suggests a translation.

21. Coordinate Geometry $\triangle MUG$ has coordinates $M(2, -4)$, $U(6, 6)$ and $G(7, 2)$. A translation maps point M to $(-3, 6)$. Find the coordinates of U' and G' under this translation.

22. Visiting Colleges Nakesha and her parents are visiting colleges. They leave their home in Enid, Oklahoma, and head for Tulsa, which is 107 mi east and 18 mi south of Enid. From Tulsa, they head to Norman, which is 83 mi west and 63 mi south of Tulsa. Where is Norman in relation to Enid? Draw a diagram to show your solution.



23. Writing Is the transformation $\triangle HYP \longrightarrow \triangle H'Y'P'$ a translation? Explain.

Find a single translation that has the same effect as each composition of translations.

24. $\langle 2, 5 \rangle$ followed by $\langle -4, 9 \rangle$

25. $\langle -3, 7 \rangle$ followed by $\langle 3, -7 \rangle$

26. $\langle 12, 0.5 \rangle$ followed by $\langle 1, -3 \rangle$

27. Coordinate Geometry $\square ABCD$ has vertices $A(3, 6)$, $B(5, 5)$, $C(4, 2)$, and $D(2, 3)$. The figure is translated so that the image of point C is the origin.

a. Find the vector that describes the translation.

b. Graph $\square ABCD$ and its image.