3-1 Reflections

What You'll Learn

- Identifying isometries
- Locating reflection images of figures

...And Why

To describe and explain the kinds of motions you encounter every day

What You'll Need

- straightedge
- protractor
- ruler
- graph paper
- MIRA™ (optional)

THINK AND DISCUSS

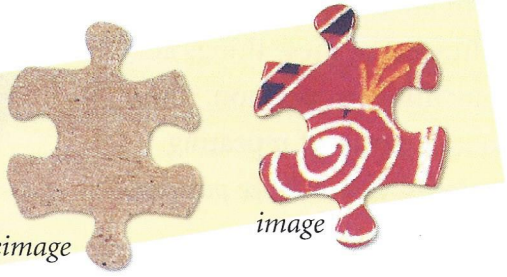
An Introduction to Transformations

Have you ever put together a jigsaw puzzle? Think about opening the box and emptying all of the puzzle pieces onto a table.

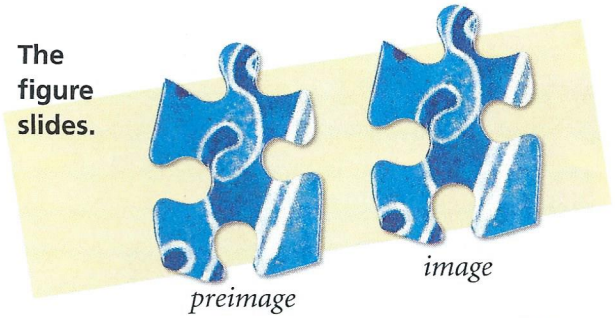
1. Describe the kinds of motions you use to put the pieces together.

You probably didn't realize it, but you use transformations when you assemble a puzzle. A **transformation** is a change in position, shape, or size of a figure. The photos below illustrate four basic transformations that you will study. Each transformed figure is the **image** of the original figure. The original figure is called the **preimage.**

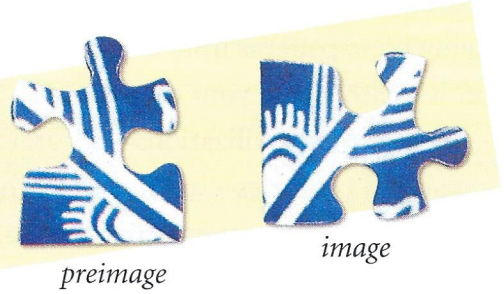
The figure flips.



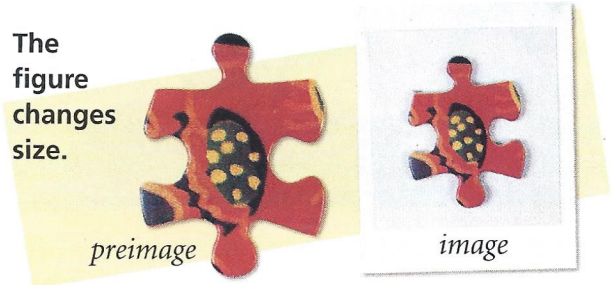
The figure slides.



The figure turns.



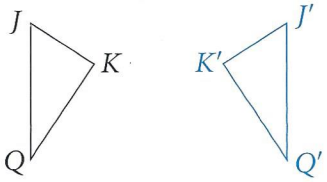
The figure changes size.



An **isometry** is a transformation in which the original figure and its image are congruent.

2. Which of the transformations shown above appear to be isometries?

A transformation **maps** a figure onto its image. An arrow ($\longrightarrow$) indicates a mapping. **Prime notation** is sometimes used to identify image points. In the diagram, K' (read "K prime") is the image of K. Notice that corresponding points of the original figure and its image are listed in the same order, just as corresponding points of congruent and similar figures are listed in the same order.



$\triangle JKQ \longrightarrow \triangle J'K'Q'$

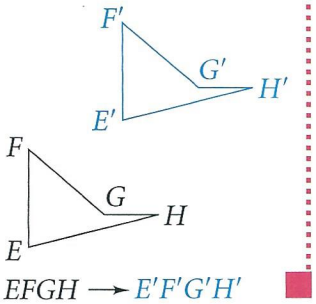
$\triangle JKQ$ maps to $\triangle J'K'Q'$.

Example 1

In the diagram, $E'F'G'H'$ is the image of $EFGH$.

a. Name the images of $\angle F$ and $\angle H$.

b. List all pairs of corresponding sides.



a. $\angle F'$ is the image of $\angle F$.
$\angle H'$ is the image of $\angle H$.

b. $\overline{EF}$ and $\overline{E'F'}$; $\overline{FG}$ and $\overline{F'G'}$;
$\overline{EH}$ and $\overline{E'H'}$; $\overline{GH}$ and $\overline{G'H'}$

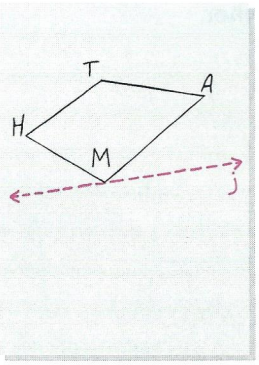
3. Which of the four types of transformations shown on the previous page is illustrated in Example 1?

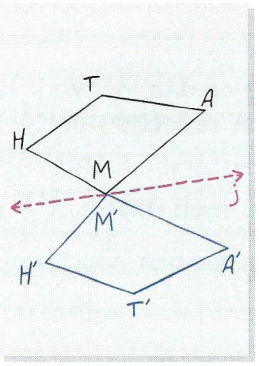
4. **Try This** List the corresponding segments and angles for the transformation $TORN \longrightarrow SAKE$.

Reflections

A flip is also known as a *reflection*. You see reflections almost every day. This morning, for example, you probably looked in the mirror before you headed out the door. In the following activity, you will investigate some properties of reflections.

WORK TOGETHER





- Have each person in your group use a straightedge to draw a quadrilateral *MATH* on the top half of a sheet of paper. Draw a line j that intersects the quadrilateral at M.
- Fold the paper along line j, then use a straightedge to trace the reflection image of *MATH* onto the bottom portion of your paper. (You could also create the image by using a MIRA™.) Label the corresponding vertices of the image M', A', T', and H'.

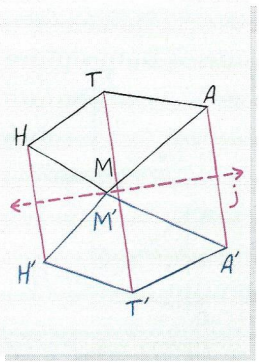
5. Measure corresponding angles and segments in *MATH* and $M'A'T'H'$. Is a reflection an isometry? Explain.

6. Make a **conjecture** about the image of a point that lies on the line of reflection. (*Hint:* Consider point M and its image.)

7. **a.** In your original figure, did you write the labels M, A, T, and H in clockwise or counterclockwise order around the quadrilateral?

b. In the reflection image, do the labels M', A', T', and H' appear in clockwise or counterclockwise order around the quadrilateral?

c. What property of reflections do parts (a) and (b) suggest?



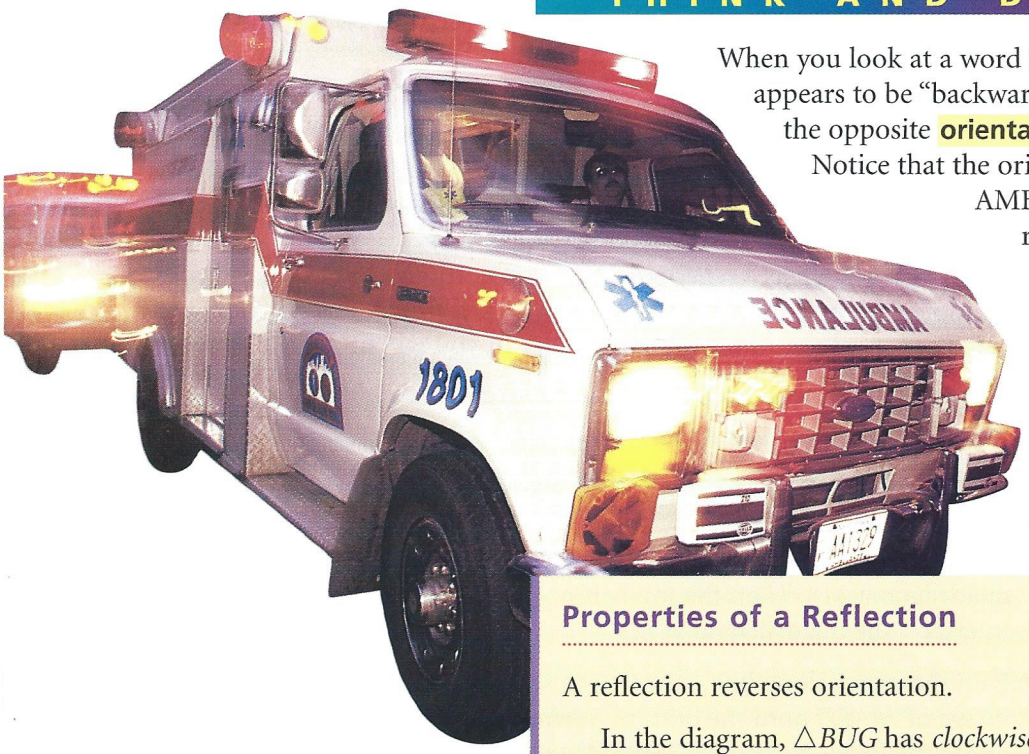
- Use a straightedge to draw segments $\overline{AA'}$, $\overline{TT'}$, and $\overline{HH'}$.

8. a. Line j divides each segment you drew into two parts. Compare the lengths of the two parts of each of the segments.

b. Line j forms four angles with each segment you drew. Use a protractor to find the measures of each of the angles.

c. Use your answers to parts (a) and (b) to complete the statement: Line j is the __?__ of the segment that connects a point and its image.

THINK AND DISCUSS



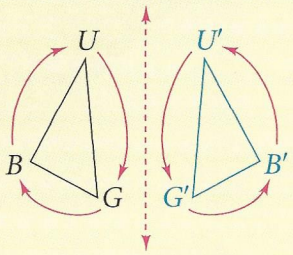
When you look at a word in a mirror, the image appears to be "backwards." The reflected word has the opposite **orientation** of the original word. Notice that the orientation of the word AMBULANCE in the photograph is reversed. The fronts of emergency vehicles often have mirror-image words on them so that drivers looking through rear-view mirrors can easily read them.

You discovered both of the following properties of reflections in the Work Together.

Properties of a Reflection

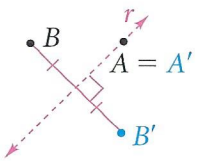
A reflection reverses orientation.

In the diagram, $\triangle BUG$ has *clockwise* orientation, so its image $\triangle B'U'G'$ has *counterclockwise* orientation.



A reflection is an isometry.

In the diagram, $\triangle BUG \cong \triangle B'U'G'$.

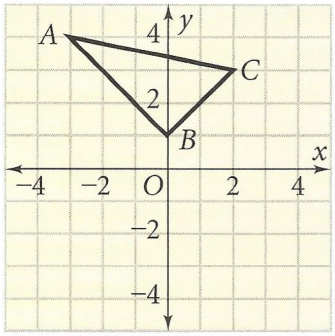


The other properties of reflections that you explored in the Work Together form the basis of the definition of a reflection. A **reflection** in line r is a transformation for which the following are true.

- If a point A is on line r, then the image of A is itself (that is, $A = A'$).
- If a point B is not on line r, then r is the perpendicular bisector of $\overline{BB'}$.

9. **Critical Thinking** Suppose you are given a point R and its reflection image R'. How could you find the line of reflection?

Example 2

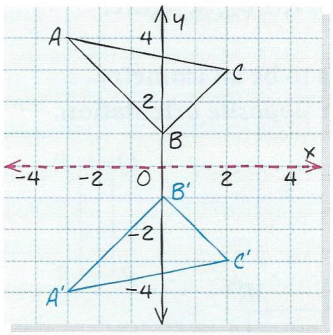


Coordinate Geometry Copy $\triangle ABC$ and draw its reflection image in each line.

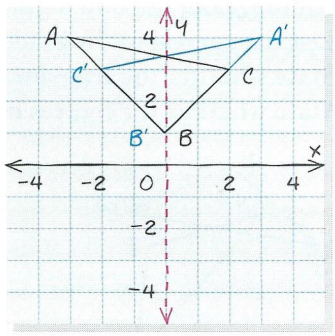
a. the x-axis **b.** the y-axis

You can find A', B', and C' by paper folding or by locating points such that the line of reflection is the perpendicular bisector of $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$.

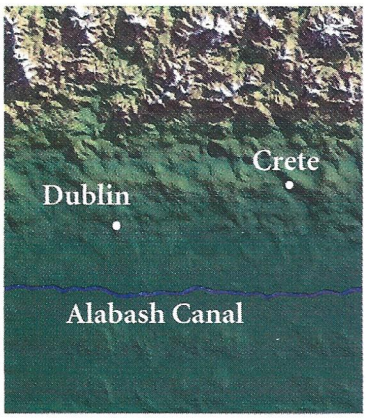
a.



b.



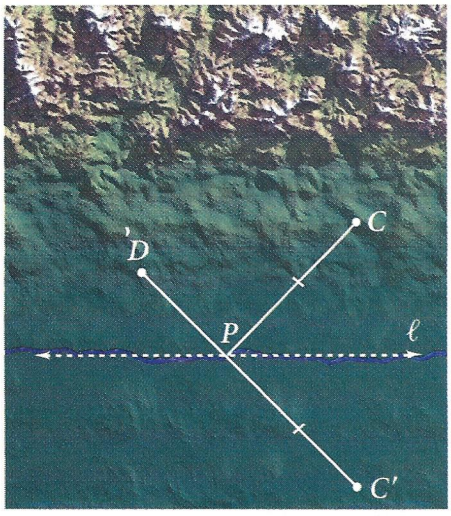
10. **Try This** Copy $\triangle ABC$ and draw its reflection image in $x = 3$.



Example 3 Relating to the Real World

Engineering The state government wants to build a pumping station along the Alabash Canal to serve the towns of Crete and Dublin. Where along the canal should the pumping station be built to minimize the amount of pipe needed to connect the towns to the pump?

You need to find the point P on ℓ such that $DP + PC$ is as small as possible. Locate C', the reflection image of C in ℓ. Because a reflection is an isometry, $PC = PC'$, and $DP + PC = DP + PC'$. The sum $DP + PC'$ is smallest when D, P, and C' are collinear. So the pump should be located at the point P where $\overline{DC'}$ intersects ℓ.

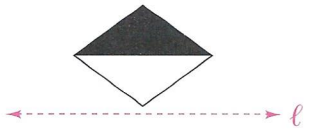


11. **Critical Thinking** Ursula began to solve Example 3 by reflecting point D in line ℓ. Will her method work? Explain.

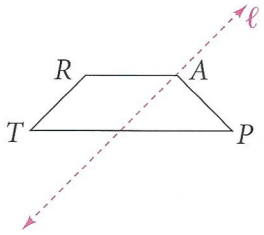
Exercises ON YOUR OWN

Copy each diagram, then find the reflection image of the figure in line ℓ.

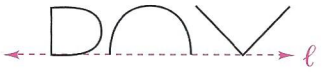
1.



2.

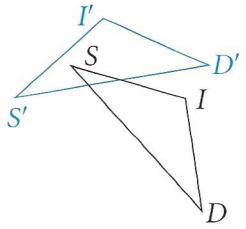


3.

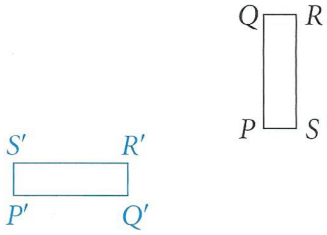


In each diagram, the blue figure is the image of the black figure.
(a) List the corresponding sides.
(b) State whether the transformation appears to be an isometry.
(c) State whether the figures have the *same* or *opposite* orientation.

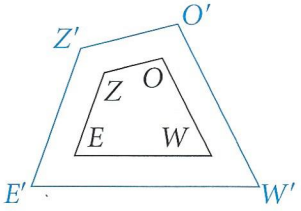
4.



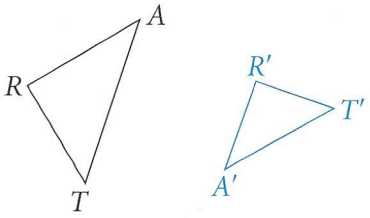
5.



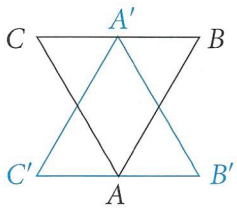
6.



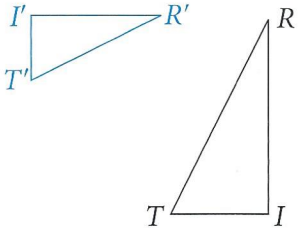
7.



8.



9.



Coordinate Geometry **Given points $J(1, 4)$, $A(3, 5)$, and $R(2, 1)$, draw $\triangle JAR$ and its reflection image in the given line.**

10. the x-axis
11. $y = 2$
12. the y-axis
13. $x = -1$
14. $y = 5$
15. $x = 2$
16. $y = -x$
17. $y = x - 3$

18. Critical Thinking Given that the transformation $\triangle ABC \longrightarrow \triangle A'B'C'$ is an isometry, list everything you know about the two figures.

19. Writing Describe an example from everyday life of a flip, a slide, a turn, and a size change.

20. Surveillance SafeCo specializes in installing security cameras in department stores. Copy the diagram onto your paper. At what point on the mirrored wall should camera C be aimed in order to photograph door D?