

What You'll Learn

- Understanding basic terms of geometry
- Understanding basic postulates of geometry

...And Why

To lay the foundation for your study of geometry

What You'll Need

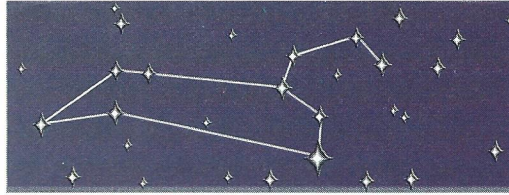
- ruler

1-2

Points, Lines, and Planes

WORK TOGETHER

Many constellations are named after animals and mythological figures. It takes some imagination to connect the points representing the stars so that the result is a recognizable figure such as Leo the Lion. There are many different ways to connect the points. How many different lines could be used to connect all ten points?



Ten major stars make up the constellation called Leo the Lion.

Work in groups of three. Make a table and look for a pattern to answer the following questions.

1. Put three points on a circle. Now connect the three points with as many lines as possible. How many lines do you need?
2. Put four points on another circle. How many lines can you draw connecting four points?
3. Repeat for five points on a circle and then for six points. How many lines can you draw to connect the points?
4. Use inductive reasoning to tell how many lines you could draw to connect the ten points of the constellation Leo the Lion.

THINK AND DISCUSS

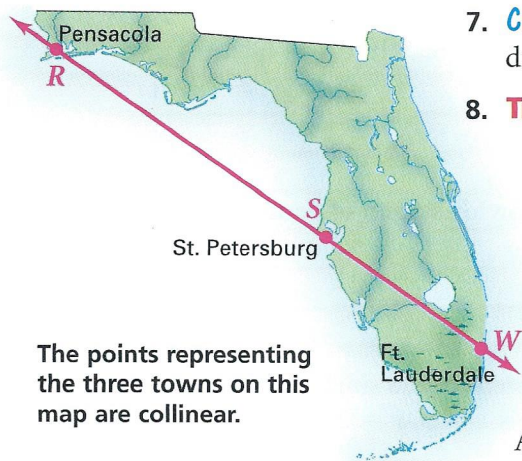
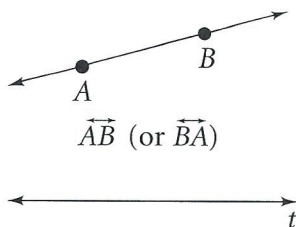
P ●
point **P**

Basic Terms

Since stars are so far away, they appear quite small to us. We think of them as points even though they are actually quite large. In geometry a **point** has no size.

You can think of it as a location. A point is represented by a small dot and is named by a capital letter. All geometric figures are made up of points. **Space** is the set of all points.

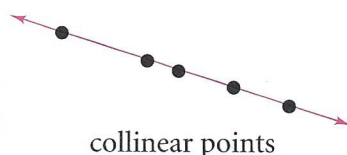
5. **Open-ended** Name something in your classroom that is a physical representation of a point.



You can think of a **line** as a series of points that extends in two opposite directions without end. You can name a line by two points on the line, such as \overleftrightarrow{AB} (read "line AB"). Another way to name a line is with a single lowercase letter, such as line t .

6. **Open-ended** Describe some physical representations of lines in the real world.
7. **Critical Thinking** Why do you think arrowheads are used when drawing a line or naming a line such as \overleftrightarrow{AB} ?
8. **Try This** Name the line at the left in as many ways as possible.

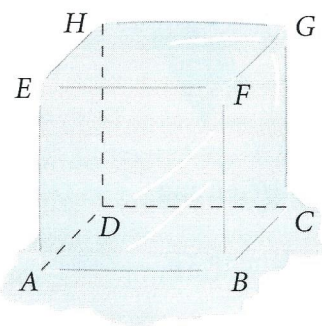
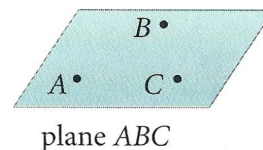
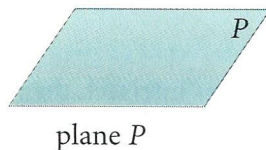
Points that lie on the same line are **collinear**.



A **plane** is a flat surface that extends in all directions without end. It has no thickness.

9. **Open-ended** Name three objects in your classroom that represent planes.

You can name a plane either by a single capital letter or by naming at least three noncollinear points in the plane.



In the diagram, each surface of the ice cube is part of a plane.

10. How many planes are suggested by the surfaces of the ice cube?
11. **Try This** Name the plane represented by the front of the ice cube in several different ways.

Points and lines in the same plane are **coplanar**.

12. **Try This** Name a point that is coplanar with the given points.
 - a. E, F, G
 - b. B, C, G
 - c. A, D, E
 - d. D, C, G
13. **Try This** Name two lines that are coplanar with \overleftrightarrow{AB} and \overleftrightarrow{DC} .

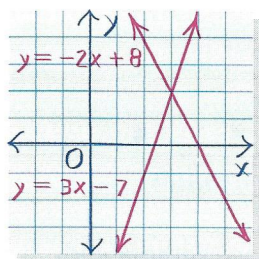
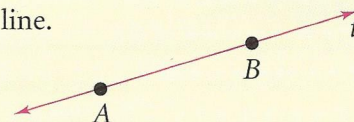
Basic Postulates

A **postulate** is an accepted statement of fact. You used some of the following geometry postulates in algebra. For example, when you graphed an equation such as $y = -2x + 8$, you began by plotting two points and then you drew the line through those two points.

Postulate 1-1

Through any two points there is exactly one line.

Line t is the only line that passes through points A and B .



In algebra, one way to solve the following system of equations is to graph the two equations.

$$\begin{aligned} y &= -2x + 8 \\ y &= 3x - 7 \end{aligned}$$

As the graph shows, the two lines intersect at a single point, $(3, 2)$. The solution to the system of equations is $x = 3$, $y = 2$. This illustrates the following postulate.

Postulate 1-2

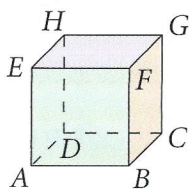
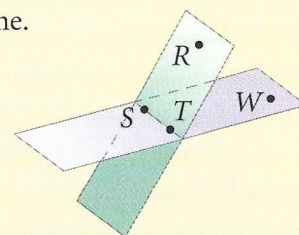
If two lines intersect, then they intersect in exactly one point.

14. **Open-ended** Describe two planes in your classroom that intersect. Also describe the intersection of the planes.

Postulate 1-3

If two planes intersect, then they intersect in a line.

Plane RST and plane STW intersect in \overleftrightarrow{ST} .



15. **a. Try This** What is the intersection of plane $HGFE$ and plane $BCGF$?
b. What is the intersection of plane AEF and plane BCG ?

A three-legged stool will always be stable, as long as the feet of the stool don't lie on a line. This illustrates the following postulate.

Postulate 1-4

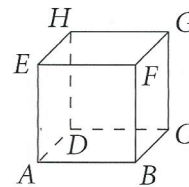
Through any three noncollinear points there is exactly one plane.



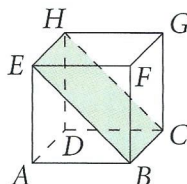
Euclid is known for compiling all the geometry of his time into postulates and theorems. His masterwork *The Elements* (about 300 B.C.) is the basis for geometry books today.

Example

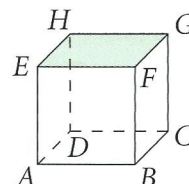
Are points E, H, B , and C coplanar?
Are points E, H, F , and B coplanar?



Yes, the plane that contains the three noncollinear points E, H , and B also contains C .



No, points E, H , and F lie in exactly one plane, which doesn't contain B .



Exercises ON YOUR OWN

Are the points collinear?

1. A, D, E
2. B, C, D
3. B, C, F
4. A, E, C
5. F, B, D

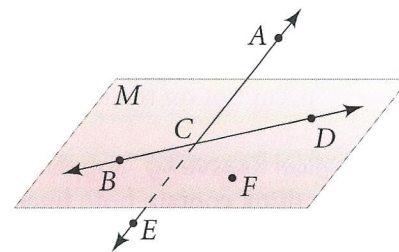
Are the points coplanar?

6. B, C, D, F
7. A, C, D, F
8. B, D, E, F
9. A, C, E, F

10. Name plane M in another way.

11. What is the intersection of plane M and \overleftrightarrow{AE} ?

12. What is the intersection of \overleftrightarrow{AE} and \overleftrightarrow{BD} ?



Exs. 1 – 12

Are the following coplanar?

13. Q, V, R
14. X, V, R
15. U, V, W, S
16. W, V, Q, T
17. point $X, \overleftrightarrow{QT}$
18. \overleftrightarrow{RS} , point X
19. $\overleftrightarrow{XW}, \overleftrightarrow{UV}$
20. $\overleftrightarrow{UX}, \overleftrightarrow{WS}$
21. $\overleftrightarrow{UV}, \overleftrightarrow{WS}$

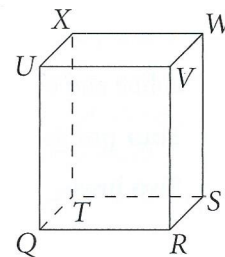
22. What is the intersection of plane $QRST$ and plane $RSWV$?

23. What is the intersection of \overleftrightarrow{UV} and plane $QTXU$?

24. Name three lines that intersect at point S .

25. Name two planes that intersect at \overleftrightarrow{TS} .

26. Name another point that is in the same plane as points Q, T , and W .



Exs. 13 – 26

27. **Writing** Surveyors and photographers use a *tripod*, or three-legged stand, for their instruments. Use one of the postulates to explain why.

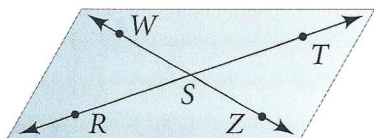
28. **Research** Find out more about Euclid's book *The Elements*. What made it such a significant book? Where did Euclid get his information?

29. How many planes contain three collinear points? Explain.

30. Which postulate is sometimes stated as "Two points determine a line"?

31. **Standardized Test Prep** Which of the following is *not* an acceptable name for the plane shown?

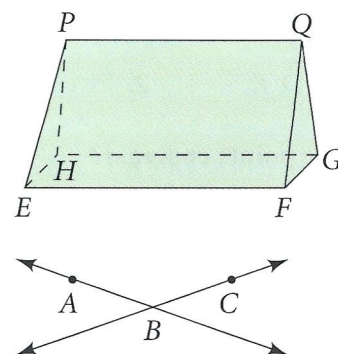
- A. plane RSZ
- B. plane $RSWZ$
- C. plane WSZ
- D. plane $RSTW$
- E. plane STZ



32. How many planes contain each line and point?

- a. \overleftrightarrow{EF} and point Q
- b. \overleftrightarrow{PH} and point E
- c. \overleftrightarrow{FG} and point P
- d. \overleftrightarrow{EP} and point G
- e. Use inductive reasoning. What do you think is true of a line and a point not on the line?

33. **Logical Reasoning** Suppose two lines intersect. How many planes do you think contain both lines? Use the diagram at the right to explain your answer.

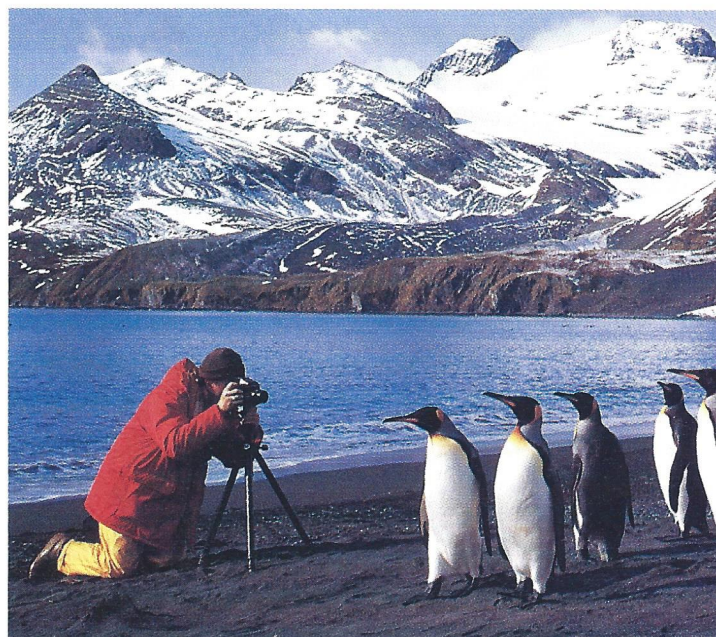
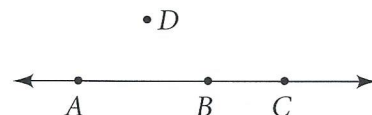


Complete with *always*, *sometimes*, or *never* to make a true statement.

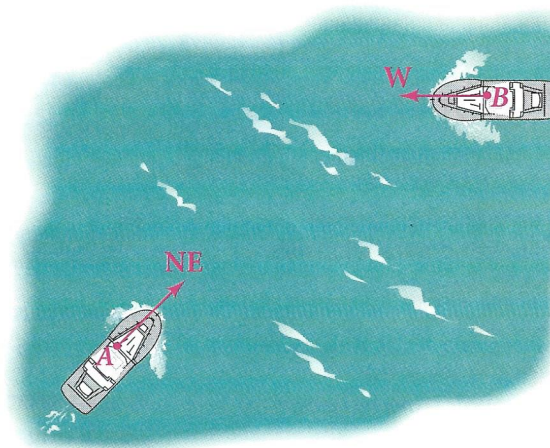
- 34. Intersecting lines are ? coplanar.
- 35. Two planes ? intersect in exactly one point.
- 36. Three points are ? coplanar.
- 37. A line and a point not on the line are ? coplanar.
- 38. Four points are ? coplanar.
- 39. Two lines ? meet in more than one point.

Probability Given points A , B , C , and D as shown, solve each problem.

- 40. Two points are picked at random. Find $P(\text{they are collinear})$.
- 41. Three points are picked at random.
 - a. Find $P(\text{they are collinear})$.
 - b. Find $P(\text{they are coplanar})$.



42. **Navigation** Rescue teams use the principles in Postulates 1-1 and 1-2 to determine the location of a distress signal. In the diagram, a ship at point A receives a signal from the northeast. A ship at point B receives the same signal from due west. Trace the diagram and find the location of the distress signal. Explain how the two postulates help to find the location of the distress signal.

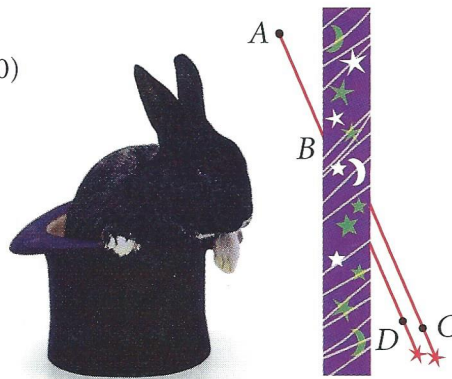


Coordinate Geometry Are the points collinear? Graph them to find out.

43. (1, 1), (4, 4), (-3, -3) 44. (2, 4), (4, 6), (0, 2)
45. (0, 0), (8, 10), (4, 6) 46. (0, 0), (0, 3), (0, -10)

47. **Open-ended** Give an example from your classroom or your home of three planes intersecting in one line.

48. **Optical Illusions** The diagram at the right is an optical illusion. Which points are collinear, A, B, C or A, B, D? Are you sure? Use a ruler to check your answer.



Exercises MIXED REVIEW

Algebra Evaluate each expression for the given values.

49. $a^2 + b^2$ for $a = 3$ and $b = -5$ 50. $\frac{1}{2}bh$ for $b = 8$ and $h = 11$
51. $2\ell + 2w$ for $\ell = 3$ and $w = 7$ 52. $b^2 - 4ac$ for $a = 2$, $b = 5$, and $c = 1$

53. **Patterns** What is the last digit of 3^{45} ? Make a table and use inductive reasoning. Explain the pattern.

For more practice with evaluating expressions, see Skills Handbook page 659.

Getting Ready for Lesson 1-3

Will the lines intersect or not?

