Congruent and Similar Figures

What You'll Learn

• Measuring congruent and similar figures

• Using properties of congruence and similarity

...And Why

To model real-world situations, such as mass production and photography

What You'll Need

• scissors
• protractor
• centimeter ruler
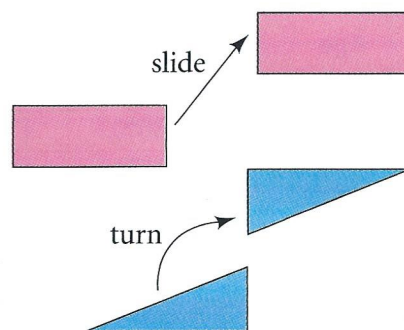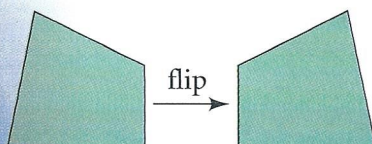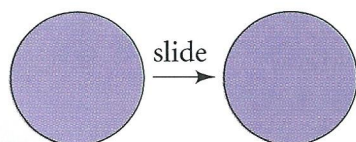
THINK AND DISCUSS

Congruent Figures

Congruent figures have exactly the same size and shape. When two figures are congruent you can slide, flip, or turn one so that it fits exactly on the other one.


Congruent circles have congruent radii. **Congruent polygons** have congruent corresponding parts. The matching angles and sides of congruent polygons are called *corresponding parts*. Matching vertices are *corresponding vertices*. When you name congruent polygons, always list corresponding vertices in the same order.

C corresponds to R.

$\angle B$ corresponds to $\angle Q$.

$\overline{AX}$ corresponds to $\overline{PY}$.

$ACBX \cong PRQY$
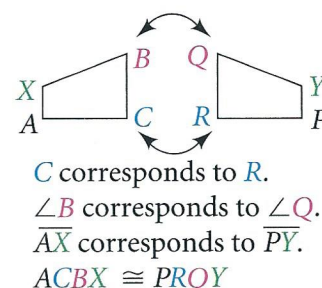
Example 1 **Relating to the Real World** 🌐

Rocketry The fins of the rocket are congruent pentagons. Find $m\angle B$.

Because the fins are congruent, $\angle B \cong \angle E$. So you can find $m\angle B$ by first finding $m\angle E$.
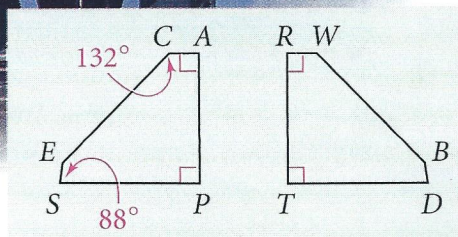
By the Polygon Interior Angle-Sum Theorem, you know that the sum of the measures of the interior angles of pentagon $SPACE$ is $(5 - 2)180$, or 540.

$$m\angle S + m\angle P + m\angle A + m\angle C + m\angle E = 540$$
$$88 + 90 + 90 + 132 + m\angle E = 540$$
$$400 + m\angle E = 540$$
$$m\angle E = 140$$
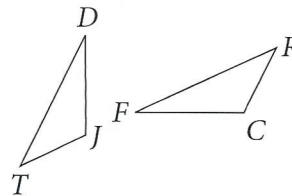
So $m\angle E = m\angle B = 140$.

Example 2

$\triangle TJD \cong \triangle RCF$. List congruent corresponding parts.

$\overline{TJ} \cong \overline{RC}$ $\overline{JD} \cong \overline{CF}$ $\overline{DT} \cong \overline{FR}$

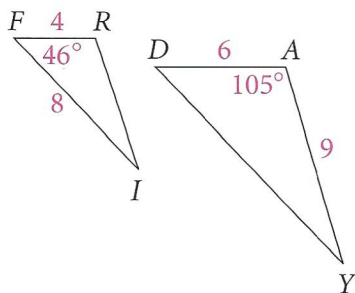$\angle T \cong \angle R$ $\angle J \cong \angle C$ $\angle D \cong \angle F$

1. **Try This** $\triangle WYS \cong \triangle MKV$. List congruent corresponding parts.

Similar Polygons

Two figures that have the same shape but not necessarily the same size are similar ($\sim$). Two polygons are **similar** if (1) corresponding angles are congruent and (2) corresponding sides are proportional. The ratio of the lengths of corresponding sides is the **similarity ratio.**

Example 3

$\triangle FRI \sim \triangle DAY$. Find:

a. the similarity ratio   b. $m\angle R$ c. RI

a. Since $\overline{FR}$ and $\overline{DA}$ are corresponding sides, the similarity ratio is $\frac{FR}{DA} = \frac{4}{6} = \frac{2}{3}$.

b. $\angle R$ corresponds to $\angle A$, so $m\angle R = m\angle A$. $m\angle A = 105$, so $m\angle R = 105$.

c. Write a proportion to solve for RI.

$$\frac{FR}{DA} = \frac{RI}{AY}$$

$\frac{4}{6} = \frac{RI}{9}$ Substitution

$36 = 6 \cdot RI$ Use cross-products.

$6 = RI$ Divide each side by 6.

2. **Try This** Find DY and $m\angle D$.

3. a. Find the perimeter of $\triangle FRI$.
   b. Find the perimeter of $\triangle DAY$.
   c. What is the ratio of the perimeter of $\triangle FRI$ to the perimeter of $\triangle DAY$?
   d. Compare your answer to part (c) to the similarity ratio. Make a **conjecture** about the ratio of the perimeters of similar figures.
   e. Test your **conjecture** by drawing and measuring other pairs of similar polygons.

4. **Critical Thinking** What type of similar figures have a similarity ratio of 1?

Example 4   **Relating to the Real World** 🌐

Photography   You want to enlarge a photo that is 4 in. tall and 6 in. wide into a poster. The poster will be 24 in. wide. How tall will it be?

$$\frac{6}{24} = \frac{4}{x} \qquad \textcolor{blue}{\textbf{Write a proportion.}}$$

$$6 \cdot x = 24 \cdot 4 \qquad \textcolor{blue}{\textbf{Use cross-products.}}$$

$$x = 16 \qquad \textcolor{blue}{\textbf{Divide each side by 6.}}$$

The poster will be 16 in. tall.

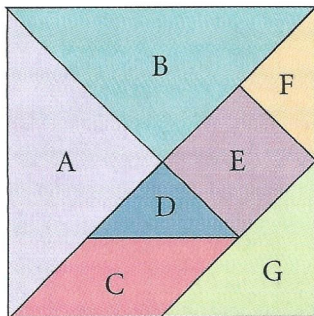24 in.

x

4 in.

6 in.

5. What is the similarity ratio of the photo to the poster?

6. Critical Thinking A school photo package comes with an 8 in.-by-10 in. photo and a 5 in.-by-7 in. photo. Are the photos similar? Explain.


W O R K   T O G E T H E R

Trace the diagram and cut out the seven pieces. (Or use the tans you made in Lesson 2-1.)
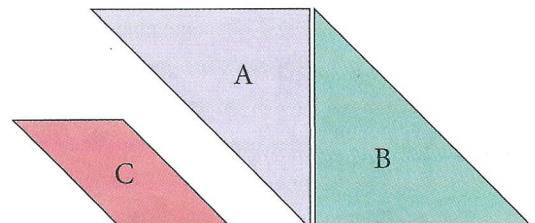
7. Which pieces are congruent?

8. Which pieces are similar? (Check that all pairs of corresponding angles are congruent and that corresponding sides are proportional.)

9. Find other congruent and similar pairs by placing pieces together. Record your answers.
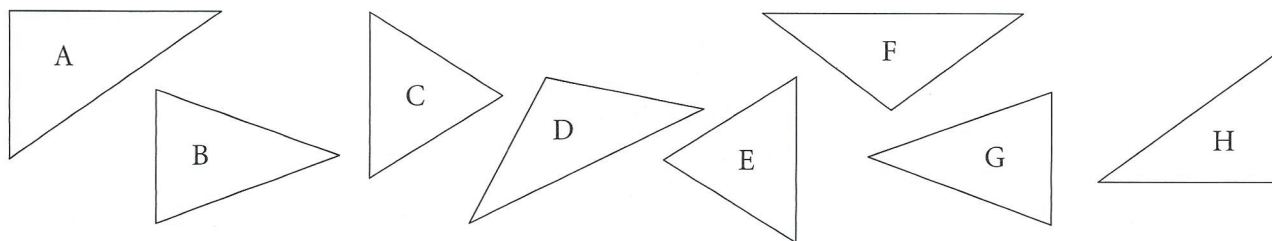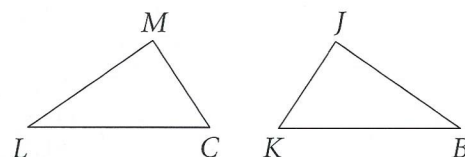
Here are some examples:

congruent   similar

B   F   A   E   D   C   G

F   D   E   A   C   B
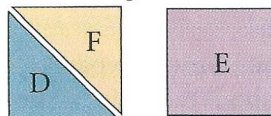
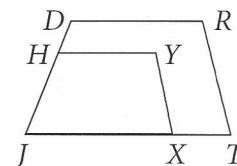1. Identify the pairs of triangles that appear to be congruent.

△**LMC** ≅ △**BJK**. Complete the congruence statements.

2. $\overline{LC} \cong$ ■   3. $\overline{KJ} \cong$ ■   4. $\overline{JB} \cong$ ■

5. $\angle L \cong$ ■   6. $\angle K \cong$ ■   7. $\angle M \cong$ ■

8. $\triangle CML \cong$ ■   9. $\triangle KBJ \cong$ ■   10. $\triangle MLC \cong$ ■

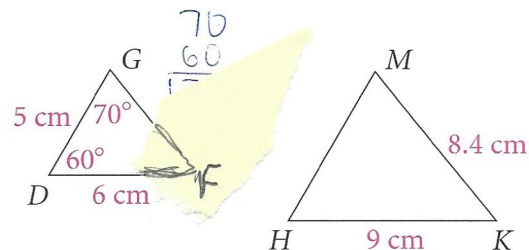**JDRT ~ JHYX**. Complete the proportions and congruence statements.

11. $\dfrac{JD}{JH} = \dfrac{DR}{■}$ 12. $\dfrac{RT}{YX} = \dfrac{■}{JX}$ 13. $\dfrac{■}{DR} = \dfrac{YX}{RT}$

14. $\angle D \cong$ ■   15. $\angle Y \cong$ ■   16. $\angle T \cong$ ■

△**DFG ~** △**HKM**. Use the diagram to find the following.

17. the similarity ratio of △DFG to △HKM

18. the similarity ratio of △HKM to △DFG

19. $m\angle F$ 20. $m\angle K$ 21. $m\angle M$

22. $\dfrac{DF}{HK}$ 23. HM 24. GF

In Exercises 25–27, **POLY** ≅ **SIDE**.

25. List four pairs of congruent angles.

26. List four pairs of congruent sides.

27. Complete the congruence statements.
   a. $OLYP \cong$ ■   b. $DESI \cong$ ■

28. **Art** An art class is painting a mural for a spring festival. The students are working from a diagram that is 48 in. long and 36 in. high. Find the length of the mural if its height is to be 12 ft.

29. **Research** A *fractal* is a self-similar geometric pattern. It is made up of parts that are similar to the object itself. A good example of this is a fern frond. Break off any leaflet and the leaflet looks like a small fern frond. Investigate fractals and **summarize** your findings.
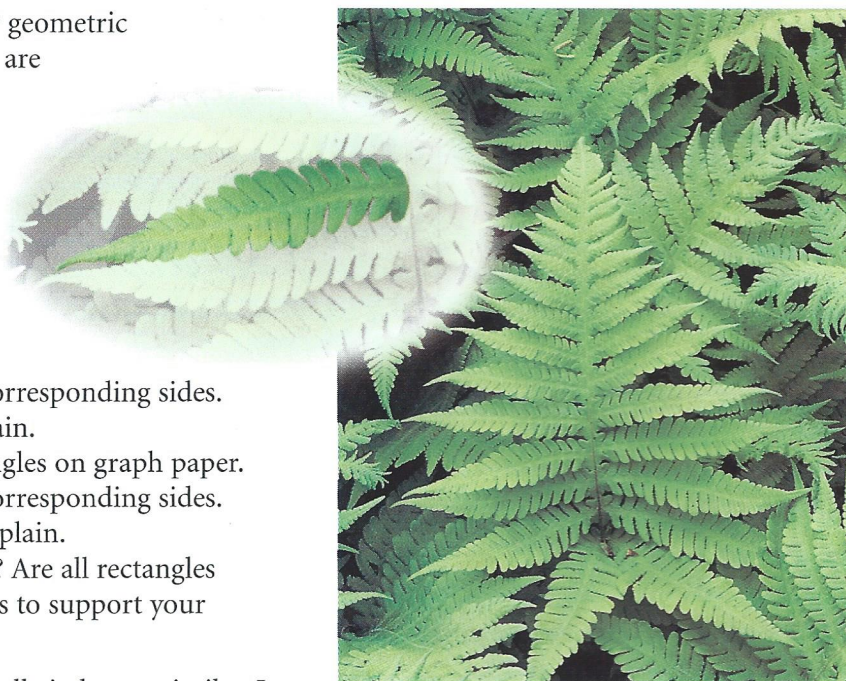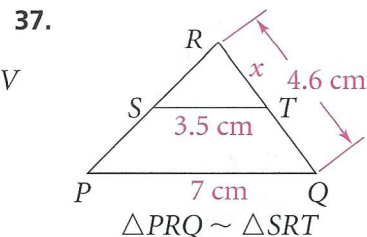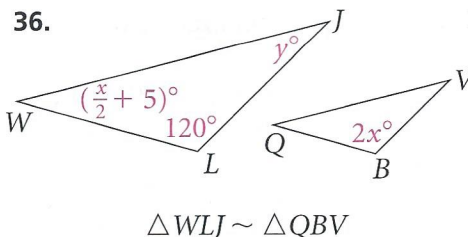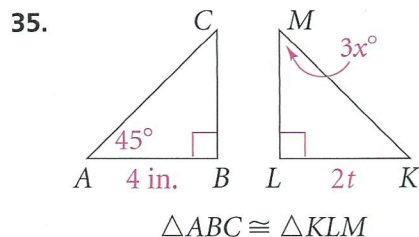
30. **a.** Draw two different-sized squares on graph paper. Find the ratio of each pair of corresponding sides.
    b. Are these squares similar? Explain.
    c. Draw two different-sized rectangles on graph paper. Find the ratio of each pair of corresponding sides.
    d. Are these rectangles similar? Explain.
    e. **Writing** Are all squares similar? Are all rectangles similar? Explain. Sketch pictures to support your conclusion.

31. **Critical Thinking** Kimi claims that all circles are similar. Is she right? Explain.

Write a congruence statement for each pair of triangles.

32.

33. *E* is the midpoint of $\overline{CD}$.

34. $\overrightarrow{TK}$ bisects $\angle PTR$.
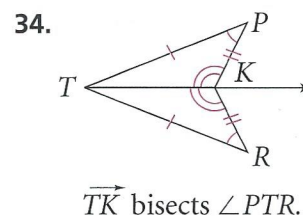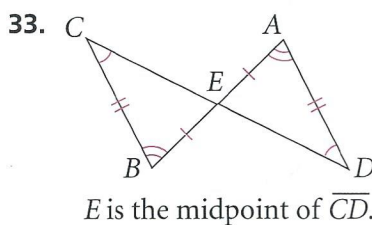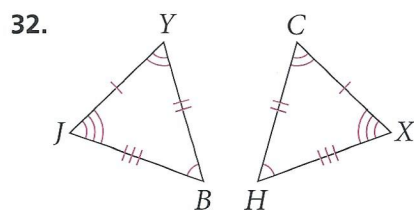
📓 **Choose** Use pencil and paper, mental math, or a calculator to find the values of the variables.

35. $45°$ 4 in. $3x°$ $2t$
    $\triangle ABC \cong \triangle KLM$

36. $y°$ $\left(\frac{x}{2}+5\right)°$ $120°$ $2x°$
    $\triangle WLJ \sim \triangle QBV$

37. x 4.6 cm 3.5 cm 7 cm
    $\triangle PRQ \sim \triangle SRT$

38. **Standardized Test Prep** $\triangle KJH$ is congruent to the triangle shown. Which of these *cannot* be the coordinates of point *H*?
    A. $(5, 0)$ **B.** $(5, 4)$ **C.** $(6, 4)$
    D. $(6, 0)$ **E.** $(7, 0)$